BALASORE SCHOOL OF ENGINEERING, BALASORE

STUDY MATERIAL FOR MATHEMATICS (TH.-3)

SECTION :- A,B,C,D

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CHAPTER – 1 (VECTOR ALGEBRA)

[2016,Q1(IV)]

01.(a) Find the value of λ if $\vec{a} = (2, -2, 1)$ and $\vec{b} = (0, 2\lambda, 1)$ are perpendicular

Soln. We know that it two vectors are $\perp$ then $\vec{a}, \vec{b} = 0$

$\Rightarrow 2.0 + (-2).2\lambda + 1.1 = 0$

$\Rightarrow 0 - 4\lambda + 1 = 0 \qquad \Rightarrow -4\lambda = -1 \quad \Rightarrow \lambda = \frac{1}{4}$

(b) Find the unit vectors ⊥ɤ to the vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{k}$ [2019(w)Q1.(H)N]

Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{i} - \hat{k}$

$$\therefore \hat{n} = \frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|}$$

$$\text{Now } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{vmatrix}$$

$= \hat{i}(-1-0) - \hat{j}(-1,0) + \hat{k}(0-1)$

$= -\hat{i} + \hat{j} - \hat{k}$

and $\vec{a} \times \vec{b} = \sqrt{(-1)^2 + 1^2 + (-1)^2} \qquad \sqrt{1+1+1} = \sqrt{3}$

so $\hat{n} = \dfrac{-\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$ (Ans).

(d) If the position vector $\vec{a}$ of a point (12, n) is such that $\left|\vec{a}\right| = 13$. Find the value of n.

[2015(w)Q.1(iii)]

Soln. Here $\vec{a}$ $12\hat{i} + n\hat{j}$

Here $\vec{a} = 12\hat{i} + n\hat{j}$

so $\left|\vec{a}\right| = \sqrt{(12)^2 + n^2} = \sqrt{144 + n^2}$

$\Rightarrow 13 = \sqrt{144 + n^2}$

$\Rightarrow 169 = 144 + n^2$

$\Rightarrow n^2 = 169 - 144 = 25$

$\Rightarrow n = \sqrt{25} = \pm 5$ (Ans)

2(a) Find the value of λ, so that the three vectors [2014(w)Q.5(b)]

$$\hat{i}+2\hat{j}+3\hat{k},\ 4\hat{i}+\hat{j}+\lambda\hat{k} \text{ and } \lambda\hat{i}-4\hat{j}+\hat{k} \text{ are co-planar.}$$

Soln. Given $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$

$\vec{b}=4\hat{i}+\hat{j}+\lambda\hat{k}$

$\vec{c}=\lambda\hat{i}-4\hat{j}+\hat{k}$

If $\vec{a},\vec{b},\vec{c}$ are co-planar then $\left[\vec{a},\vec{b},\vec{c}\right]=0$

$$\text{i.e.} \begin{vmatrix} 1 & 2 & 3 \\ 4 & 1 & \lambda \\ \lambda & -4 & 1 \end{vmatrix}=0$$

$\Rightarrow 1(1+4\lambda) - 2(4-\lambda^2) + 3(-16-\lambda) = 0$

$\Rightarrow 1 + 4\lambda - 8 + 2\lambda^2 - 48 - 3\lambda = 0$

$\Rightarrow 2\lambda^2 + \lambda - 55 = 0$

$\Rightarrow 2\lambda^2 + 11\lambda - 10\lambda - 55 = 0$

$\Rightarrow (2\lambda+11)(\lambda-5) = 0$

$\Rightarrow 2\lambda + 11 = 0$ or $\lambda - 5 = 0$

$\Rightarrow \lambda = -11/2$ or $\lambda = 5$

Thus $\lambda = 5$; the vectors are co-planar

(b) Find the area of the triangle whose two adjacent sides are determined by the vectors

$\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, and $\vec{b}=3\hat{i}-2\hat{j}+\hat{k}$ **[**2015 (w) Q.5(b)]

Soln. We know the area of the triangle whose two adjacent sides are determined by the vectors

$\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, and $\vec{b}=3\hat{i}-2\hat{j}+\hat{k}$ is

$$\frac{1}{2}\left|\vec{a}\times\vec{b}\right|$$

$$\vec{a}\times\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ +3 & -2 & 1 \end{vmatrix}$$

$\hat{i}(2+6)-\hat{j}(1+9)+\hat{k}(-2+6)$

$=8\hat{i}+10\hat{j}+4\hat{k}$

$\therefore \left|\vec{a}\times\vec{b}\right|=\sqrt{8^2+(-10)^2+4^2}=\sqrt{64+100+16}=\sqrt{180}$

$\Rightarrow \left|\vec{a}\times\vec{b}\right|=6\sqrt{5}$

But the area is $\frac{1}{2}\left|\vec{a}\times\vec{b}\right|=\frac{1}{2}.6\sqrt{5}=3\sqrt{5}$

(c) Find the angle between the vectors $5\hat{i}+3\hat{j}+4\hat{k}$ and $6\hat{i}-8\hat{j}-\hat{k}$ [2019(S) Q.7N]

Soln. Let $\vec{a}=5\hat{i}+3\hat{j}+4\hat{k}$

and $\vec{b}=6\hat{i}-8\hat{j}-\hat{k}$

Let θ be the angle between the no.

$$\text{so } \theta=\cos^{-1}\left[\frac{\vec{a}\,.\,\vec{b}}{\left|\vec{a}\right|\ \left|\vec{b}\right|}\right]$$

$$\Rightarrow \qquad \theta=\cos^{-1}\left[\frac{\left(5\hat{i}+3\hat{j}+4\hat{k}\right).\left(6\hat{i}-8\hat{j}-\hat{k}\right)}{\sqrt{5^2+3^2+4^2}.\sqrt{6^2(-8)^2(-1)^2}}\right]$$

$$=\cos^{-1}\left[\frac{\left(5\hat{i}+3\hat{j}+4\hat{k}\right).\left(6\hat{i}-8\hat{j}-\hat{k}\right)}{\sqrt{50}.\sqrt{101}}\right]$$

$$=\cos^{-1}\left[\frac{30\left(\hat{i},\hat{i}\right)+(-24)\left(\hat{j}.\hat{j}\right)+(-4)\left(\hat{k}.\hat{k}\right)}{\sqrt{50}.\sqrt{101}}\right]$$

$$=\cos^{-1}\left(\frac{30-24-4}{\sqrt{50}.\sqrt{101}}\right)=\cos^{-1}\left(\frac{2}{\sqrt{50}.\sqrt{101}}\right)$$

$$=\cos^{-1}\left(\frac{2}{5\sqrt{2}.\sqrt{101}}\right)=\cos^{-1}\left(\frac{\sqrt{2}}{5.\sqrt{101}}\right)$$ (Ans).

(3) [2016 (1.b)]

Find the scalar value of λ if $\vec{a}=(2,-2,1)$

$\vec{b}=(0,2\lambda,1)$ are perpendicular

soln. Let $\vec{a}\perp\vec{b}=\vec{a}.\vec{b}=0$

$\Rightarrow a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow 2x0+(-2).2\lambda+1.1=0$

$\Rightarrow -4\lambda=-1 \quad \Rightarrow \lambda=\dfrac{1}{4}$ (Ans).

Q: Find the scalar and vector projections of $\hat{i}-\hat{j}-\hat{k}$ on $3\hat{i}+\hat{j}+3\hat{k}$.

soln. Let $\vec{a}=\hat{i}-\hat{j}-\hat{k}$ and [2019W,2F(N)]

$\vec{b}=3\hat{i}-\hat{j}-3\hat{k}$

$$\text{scalar Projection of } \vec{a} \text{ on } \vec{b} = \left(\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right)$$

$$=\frac{\left(\hat{i}-\hat{j}-\hat{k}\right).\left(3\hat{i}-\hat{j}-3\hat{k}\right)}{\sqrt{3^2+1^2+3^2}}$$

$$=\frac{3-1-3}{\sqrt{19}}=\frac{-1}{\sqrt{19}}$$

$$\text{vector projection of } \vec{a} \text{ on } \vec{b} = \left(\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right)\frac{\vec{b}}{|\vec{b}|}$$

$$=\frac{-1}{\sqrt{19}}\cdot\frac{3\hat{i}-\hat{j}-3\hat{k}}{\sqrt{3^2+1^2+3^2}}=\frac{-1}{\sqrt{19}}\cdot\frac{3\hat{i}-\hat{j}-3\hat{k}}{\sqrt{19}}$$

$$=-\left(\frac{3\hat{i}-\hat{j}-3\hat{k}}{19}\right)$$

$$=-\left(\frac{3}{19}\hat{i}+\frac{1}{19}\hat{j}+\frac{3}{19}\hat{k}\right) \quad \text{(Ans).}$$

(5) Determine the sine angle between the vectors [2019,Q- 6(N)]

$\vec{a}=\hat{i}-3\hat{j}+\hat{k}$

$\vec{b}=\hat{i}+\hat{j}+\hat{k}$

soln. $$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 1\\ 1 & 1 & 1\end{vmatrix}$$

$$=\hat{i}(-3-1)-\hat{j}(1-1)+\hat{k}(1+3)$$

$$=-4\hat{i}+4\hat{k}$$

$$|\vec{a}\times\vec{b}|=\sqrt{(-4)^2+4^2}=\sqrt{32}$$

$$\sin\theta=\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}=\frac{\sqrt{32}}{\sqrt{1^2+(-3)^2+1^2}\sqrt{1^2+1^2+1^2}}$$

$$=\frac{\sqrt{32}}{\sqrt{11}\sqrt{3}}=\sqrt{\frac{32}{33}} \quad \text{(Ans).}$$

Q: Find the vector parallel to the sum of vectors [2015 3(a)]

$\vec{a} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ & $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

soln. $\vec{r} = \vec{a} + \vec{b} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

$|\vec{r}| = \sqrt{3^2 + 6^2 + (-2)^2} = 7$

A unit vector parallel to $\vec{r}$

is $\frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$.

Q: Find a unit vector perpendicular to both the vectors [2017(w) Q.2(b)]

$\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ & $\vec{b} = 3\hat{i} - \hat{j} + 3\hat{k}$

soln. $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{b} = 3\hat{i} - \hat{j} + 3\hat{k}$

Now $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 3 & -1 & 3 \end{vmatrix}$

$= \hat{i}(3-1) - \hat{j}(6+3) + \hat{k}(-2-3)$

$\Rightarrow \vec{a} \times \vec{b} = 2\hat{i} - 9\hat{j} - 5\hat{k}$

and $|\vec{a} \times \vec{b}| = \sqrt{2^2 + (-9)^2 + (-5)^2} = \sqrt{4 + 81 + 25} = \sqrt{110}$

Hence the unit vector perpendicular both $\vec{a}$ & $\vec{b}$ is given by

$\hat{n} = \frac{2\hat{i} - 9\hat{j} - 5\hat{k}}{\sqrt{110}}$ (Ans).

Q: Show that the vector $\hat{i} - 3\hat{j} + 4\hat{k}$, $2\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - 7\hat{j} + 10\hat{k}$ [2018(w) Q. 6(b)] are coplanar

soln. we know that, if three vectors are coplanar

then $[a\ b\ c] = 0$

i.e. $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

Let $\vec{a} = \hat{i} - 3\hat{j} + 4\hat{k}$ and

$\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{c} = 4\hat{i} - 7\hat{j} + 10\hat{k}$

Now $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} 1 & -3 & 4 \\ 2 & -1 & 2 \\ 4 & -7 & 10 \end{vmatrix}$

$= 1(-10 + 14) + 3(20 - 8) + 4(-14 + 4)$

$= 4 + 36 - 40 = 40 - 40 = 0$

$\therefore \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

Hence, the given vectors are coplanar (Proved)

LIMITS AND CONTINUITY

CHAPTER – 2

01 [2016,2b]

Find $\lim\limits_{x \to 1} \dfrac{x^2 - 2x + 1}{x^2 - x}$

Ans : $\lim\limits_{x \to 1} \dfrac{x^2 - 2x + 1}{x^2 - x} \left(\div \text{form}\right)$

$$= \lim_{x \to 1} \frac{(x-1)}{x(+1)} = \lim_{x \to 1} \frac{x-1}{x} = \frac{1-1}{1} = \frac{0}{1} = 0.$$

Q.

Find $\lim\limits_{x \to 0} \dfrac{a^x - b^x}{c^x - d^x}$ **[2018,2b(s)]**

Ans : $\lim\limits_{x \to 0} \dfrac{a^x - b^x}{c^x - d^x} = \lim\limits_{x \to 0} \dfrac{\frac{a^x - b^x}{x}}{\frac{c^x - d^x}{x}}$

$$= \frac{\lim\limits_{x \to 0} \frac{a^x - b^x}{x}}{\lim\limits_{x \to 0} \frac{c^x - d^x}{x}} = \frac{\lim\limits_{x \to 0} \frac{b^x \left[\frac{a^x}{b^x} - 1\right]}{x}}{\lim\limits_{x \to 0} \frac{d^x \left[\frac{c^x}{d^x} - 1\right]}{x}}$$

$$= \frac{\lim\limits_{x \to 0} b^x \times \lim\limits_{x \to 0} \frac{\left(\frac{a}{b}\right)^x - 1}{x}}{\lim\limits_{x \to 0} d^x \times \lim\limits_{x \to 0} \frac{\left(\frac{c}{d}\right)^x - 1}{x}} = \frac{b^o \times \log \frac{a}{b}}{d^o \times \log \frac{c}{d}}$$

$$= \frac{\log a - \log b}{\log c - \log d}$$

1.c Find thevalue of K such that $f(x)=\begin{cases}\dfrac{8^x-4^x-2^x+1}{x^2}, x+0\\ K,\ \text{if } x\neq 0\end{cases}$ [2017,1c(w)]

is contineous at $x=0$

Ans: $f(x)=\begin{cases}\dfrac{8^x-4^x-2^x+1}{x^2}, & \text{If } x\neq 0\\ K, & \text{if } x=0\end{cases}$

Given f(x) is contineous at $x=0$

$\Rightarrow$ **Llimiting value = functional value at x = 0**

For limiting value of f(x) at x = 0

$\lim\limits_{x\to 0} f(x),\quad x\to 0,\quad x\neq 0, f(x)=\dfrac{8^x-4^x-2^x+1}{x^2}$

$\lim\limits_{x\to 0}\dfrac{8^x-4^x-2^x+1}{x^2}\lim\limits_{x\to 0}\dfrac{4^x\left(2^x-1\right)-1\left(2^x-1\right)}{x^2}$

$\lim\limits_{x\to 0}\dfrac{\left(2^x-1\right)\left(4^x-1\right)}{x^2}=\lim\limits_{x\to 0}\left(\dfrac{2^x-1}{x}\right)\times\left(\dfrac{4^x-1}{x}\right)$

$\lim\limits_{x\to 0}\dfrac{4^x-1}{x}\times\lim\limits_{x\to 0}\dfrac{2^x-1}{x}$

= log 4 × log 2 = 2 log 2 × log 2=2(log2)^2 ------------ (1)

Functional value of f(x) at x = 0

F(0) = k - -------------------------- (2)

From (1) and (2)

K = 2 (log)^2 (Ans)

Q. **[2016(s0, 1-I,2019w,s,1a(O,N)]**

Evaluate $\lim\limits_{x\to 0}\dfrac{\sin x}{\sin bx}$

Ans:

$\lim\limits_{x\to 0}\dfrac{\sin x}{\sin bx}$

$=\lim\limits_{x\to 0}\dfrac{\sin x}{ax}\cdot\dfrac{bx}{\sin bx}\cdot\dfrac{ax}{bx}$

$=\dfrac{a}{b}\lim\limits_{x\to 0}\dfrac{\sin x}{ax}.\lim\limits_{x\to 0}\dfrac{bx}{\sin bx}$

$=\dfrac{a}{b}\times 1\times 1$

$=\dfrac{a}{b}$

Q. Evaluate $\lim_{x\to\frac{\pi}{2}} \frac{\cos^2 x}{1-\sin x}$ **[2017, 1 – i]**

Ans: $\lim_{x\to\frac{\pi}{2}} \frac{\cos^2 x}{1-\sin x}$

$= \lim_{x\to\frac{\pi}{2}} \frac{1-\sin^2 x}{1-\sin x}$

$= \lim_{x\to\frac{\pi}{2}} \frac{(1+\sin x)(1-\sin x)}{1-\sin x}$

$= \lim_{x\to\frac{\pi}{2}} (1+\sin x)$

$= 1+\sin\frac{\pi}{2}$

$= 1+1$

$= 2$

Q. Evaluate $\lim_{x\to 0} \frac{\sin^{-1} x}{x}$ **[[2016, 1 –i i]**

Ans: $\lim_{x\to 0} \frac{\sin^{-1} x}{x}$ $\left[\begin{array}{l}\text{Let } x = y \\ \Rightarrow x = \sin y \\ x \to 0 \text{ as } y \to 0\end{array}\right.$

$= \lim_{y\to 0} \frac{y}{\sin y} = 1$

Q. Evaluate $\lim_{n\to\infty} \frac{\lfloor n-1}{\lfloor n+1}$ **[2015 (s) 1-a,2019W-1b(N)]**

Ans: $= \lim_{n\to\infty} \left(\frac{\frac{\lfloor n-1}{\lfloor n+1}}{\frac{\lfloor n+1}{\lfloor n}} \right)$

$= \lim_{n\to\infty} \frac{1-\frac{1}{\lfloor n}}{1+\frac{1}{\lfloor n}}$

$= \frac{1-0}{1+0}$

$= \frac{1}{1}$

$= 1$

Q. Evaluate $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$ **[2018(s),2b]**

Ans: $= \lim\limits_{x\to 0}\frac{(1-\cos x)+(1+\cos x)}{x^2(1+\cos x)}$

$= \lim\limits_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)}$

$= \lim\limits_{x\to 0}\frac{\sin^2 x}{x^2}.\frac{1}{1+\cos x}$

$= \lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^2 \lim\limits_{x\to 0}\frac{1}{1+\cos x}$

$= 1.\frac{1}{1+\cos x}$

$= \frac{1}{1+1}$

$= \frac{1}{2}$

[2017,2b(s)]

Q. Evaluate $\lim\limits_{x\to 0}\frac{\log_e(x+1)}{\sqrt{x+1}-1}$

Ans: $= \lim\limits_{x\to 0}\frac{\log_e(x+1)\left(\sqrt{x+1}+1\right)}{\left(\sqrt{x+1}-1\right)\left(\sqrt{x+1}+1\right)}$

$= \lim\limits_{x\to 0}\frac{\log_e(x+1)\left(\sqrt{x+1}+1\right)}{x+1-1}$

$= \lim\limits_{x\to 0}\frac{\log_e(x+1)}{x}.\lim\limits_{x\to 0}\left(\sqrt{x+1}+1\right)$

$= \lim\limits_{x\to 0}\frac{\log_e(x+1)}{x}.\lim\limits_{x\to 0}\left(\sqrt{x+1}+1\right)$

$= 1.\left(\sqrt{0+1}+1\right)$

$= 2$

Q. Examine the continuity of the function f(x) given by [2016,Q-5(c)]

$$f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, \text{where } x \neq 0 \\ 0 \quad , \text{when } x = 0 \end{cases} \quad \text{at } x = 0$$

Ans: $f(o)=0$

$$\lim_{x\to 0} f(x)$$

LHL $= \lim_{x\to 0} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$

$$\lim_{x\to 0} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$$

$$\lim_{h\to 0} \frac{e^{\frac{1}{0-h}}-1}{e^{\frac{1}{0-h}}+1}$$

$$\lim_{h\to 0} \frac{e^{-\frac{1}{h}}-1}{e^{-\frac{1}{h}}+1}$$

$$\lim_{h\to 0} \frac{e^{\frac{1}{1/h}}-1}{e^{\frac{1}{1/h}}+1}$$

$$=\frac{0-1}{0+1}$$

$$=-1$$

RHL

$$\lim_{x\to 0^+} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$$

$$= \lim_{h\to 0} \frac{e^{\frac{1}{h}}-1}{e^{\frac{1}{h}}+1}$$

$$= -1$$

$$= LHL = RHL$$

Q. Test the continuity of given function at x = 1 **[2016,1(c)W]**

Where $f(x) = \begin{cases} 2x-1 & \text{if } x \geq 1 \\ X & \text{if } x < 1 \end{cases}$

Ans: f(1) = 2 × 1 −1 = 1

LHL

$$\lim_{x \to 1^{-1}} x$$

$$= \lim_{h \to -} 1 - h$$

$$= 1 - 0$$

$$= 1$$

RHL

$$\lim_{x \to 1^{+}} 2x - 1$$

$$= \lim_{h \to 0} \left[2(1 + h) - 1\right]$$

$$= \lim_{h \to 0} 2 + 2h - 1$$

$$= \lim_{h \to 0} 1 + 2h$$

$$= 1 \times 2 \times 0$$

$$= 1$$

LHL = RHL

$$\lim_{x \to 1} f(x) = f(1)$$

Hence f(x) is continuous at x = 1

Q. Find the value of 'K' such that **[2019W,2b]**

$$f(x)\begin{cases} (1 + 3x)^{\frac{1}{x}} & , \quad x \neq 0 \\ e^{k} & , \quad x = 0 \end{cases}$$ is contineous at $x = 0$

Ans: $f(0) = e^{k}$

$$= \lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} (1 + 3x)^{\frac{3}{3x}}$$

$$= \lim_{x \to 0} \left[(1 + 3x)^{\frac{1}{3x}}\right]^{3}$$

$$= \left[\lim_{x \to 0} (1 + 3x)^{\frac{1}{3x}}\right]^{3}$$

$$= e^{3}$$

Given f(x) is continuous at x = 0

$$\lim_{x\to 0} f(x) = f(0)$$

$$\Rightarrow \quad e^3 = e^k$$

$$\Rightarrow \quad k = 3$$

Q. **[2018w,1b]**

Ans: If $f(x) = \begin{cases} \dfrac{x}{|x|} & , \quad \text{when } x \neq 0 \\ L & , \quad \text{when } x = 0 \end{cases}$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0^+} (0+h)$$

$$= \lim_{h\to 0} \frac{h}{|h|}$$

$$= \lim_{h\to 0} \frac{h}{h} = 1$$

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{-h}{|h|}$$

$$= \lim_{h\to 0} \frac{-h}{h} = -1$$

$$\lim_{x\to 0}^{+} f(x) \neq \lim_{x\to 0^-} f(x)$$

so, $\lim_{x\to 0^+} f(x)$ does not exist

Hence, f(x) is discontinuous at $x = 0$

Q. **Evaluate** $\lim_{x\to 0} \dfrac{(x+9)^{\frac{3}{2}} - 27}{x}$ **[2015s,2c]**

Ans:

put $x + 9 = z^2$

$\Rightarrow x = z^2 - 9$

when $x \to 0$

$z^2 \to 3$

$$\lim_{x\to 0} \frac{(x+9)^{\frac{3}{2}} - 27}{x}$$

$$\Rightarrow \quad \lim_{x\to 0} \frac{\left(\sqrt{x+9}\right)^3 - (3)^3}{x}$$

$$\Rightarrow \quad \lim_{x\to 3} \left\{ \frac{\dfrac{(z)^3 - (3)^3}{z-3}}{\dfrac{(z)^2 - (3)^2}{z-3}} \right\}$$

$$\Rightarrow \quad \frac{\lim_{x\to 3} \dfrac{(z)^3 - (3)^3}{z-3}}{\lim_{x\to 0} \dfrac{(z)^2 - (3)^2}{z-3}}$$

$$= \quad \frac{3(3)^{3-1}}{2(3)^{2-1}} = \frac{3 \times 3^{\not{2}3}}{2 \times 3} = \frac{9}{2}$$

Q. Examine the continuity of the function f(x) at x = 0 defined by [2015s,2b]

$$f(x)\begin{cases} \dfrac{\sin 2x}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases} \quad \text{at } x = 0$$

Ans:

$$\lim_{x\to 0} \frac{\sin 2x}{x}$$

$$= \lim_{x\to 0} \frac{\sin 2x}{x} \times 2$$

$$= 2$$

Given $f(0) = 2$

$\lim_{x\to 0} f(x) = 2 = f(0)$

So the function is continuous at x = 0.

Q. Evaluate $\lim_{x\to 0} \dfrac{x - x\cos e\, x}{\sin^3 2x}$ **[2019S,2b(N)]**

Ans:

$$\lim_{x\to 0} \frac{x - x\cos e\, x}{\sin^3 2x}$$

$$\Rightarrow \lim_{x\to 0} \frac{x(1 - \cos e\, x)}{\sin^3 2x}$$

$$\Rightarrow \lim_{x\to 0} \frac{1 - \cos e\, x}{x^2} . \frac{x^3}{\sin^3 2x}$$

$$\Rightarrow \lim_{x\to 0} \frac{2\sin^2 x}{x^2} . \frac{x^3}{\sin^3 2x}$$

$$\Rightarrow 2\left(\lim_{x\to 0} \frac{\sin x}{x}\right)^2 \times \left(\frac{1}{\lim_{x\to 0} . \dfrac{\sin^3 2x}{x^3}}\right)$$

$$\Rightarrow 2(1)^2 \times \frac{1}{\lim_{x\to 0} . \dfrac{\sin^3 2x}{8x^3} \times 8}$$

$$\Rightarrow \frac{2}{8} \times \frac{1}{\lim_{x\to 0}\left(\dfrac{\sin 2x}{2x}\right)^3}$$

$$\Rightarrow \lim_{x\to 0} \frac{\sin x}{x}$$

$$\Rightarrow \frac{1}{4} \times \frac{1}{1}$$

$$\Rightarrow \frac{1}{4}$$

Q. Evaluate $\lim\limits_{x \to 1} \frac{\log(2x-1)}{x-1}$ **[2018s,2b(n)]**

Ans: $\Rightarrow \quad \lim\limits_{x \to 1} \frac{\log(2x-1)}{x-1}$

$\Rightarrow \quad \lim\limits_{x \to 1} \frac{\log(2x-2+1)}{x-1}$

Let $y = x - 1$

when $x \to 1$

$y \to 1$

$\Rightarrow \quad \lim\limits_{y \to 0} \frac{\log(2y+1)}{y}$

$\Rightarrow \quad 2 \log e \left[\lim\limits_{y \to 0} (1+2y)^{\frac{1}{2y}} \right]$

$\Rightarrow \quad 2 \log e^{e} = 2.1 = 2$

Q: **[2019W,2e(O)]**

$$\text{IOf } f(x) = \begin{cases} ax^2 + b & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ 2ax - b & \text{if } x > \end{cases}$$

Is continuous at x = 1, then find a & b

Ans:

$\lim\limits_{x \to 1} f(x) = f(1) = 1$

i.e. $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x)$

Now $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} (ax^2 + b)$

$\Rightarrow \quad a + b = 1$, clearly $f(1) = 1$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} (2ax - b)$

$\Rightarrow 2a - b = 1 \quad a + b = 2a - b$

$a = 2b$

Again $a + b = 1 \quad \Rightarrow 3b = 1 \quad \Rightarrow \quad b = \frac{1}{3}$

and $a = 2b = 2.\frac{1}{3} = \frac{2}{3}$

$\therefore \quad a = \frac{2}{3}$ & $b = \frac{1}{3}$

DERIVATIVES
CHAPTER:3

Q. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ **[2017w,5b]**

Ans: let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ (putting $x = \tan\theta$

we get

$$y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left\{\frac{2\sin^2\left(\frac{\theta}{2}\right)}{2\sin\left(\frac{\theta}{2}\right).\cos\left(\frac{\theta}{2}\right)}\right\}$$

$$= \tan^{-1}\left\{\tan\left(\frac{\theta}{2}\right)\right\}$$

$$= \frac{1}{2}\theta = \frac{1}{2}\tan^{-1}x$$

$$\frac{dy}{dx} = \frac{1}{2(1+x^2)}$$

Hence, $\frac{dy}{dx}\left\{\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)\right\}$

$$= \frac{1}{2\left(1+x^2\right)}$$

2b. Differentiate sin x from first principle. [2017 (2(b)]

Ans: let f(x)y = sin x -------------------------------------(1)

Then f(x + δx) = sin (x + δx)

$\therefore$f(x + δx) – F(x) = sin (x +δx) – sin x

$$\lim_{\delta x\to 0}\frac{F(x+\delta x)-f(x)}{\delta x}=\lim_{\delta x\to 0}\frac{\sin(x+\delta x)-\sin x}{\delta x}$$

$$\Rightarrow \quad F'x=\lim_{\delta x\to 0}\frac{2\cos\frac{x+\delta x+x}{2}.\sin\frac{x+\delta x+x}{2}}{\delta x}$$

$$= \quad \lim_{\delta x\to 0}\frac{2\cos\frac{2x+\delta x}{2}.\sin\frac{\delta x}{2}}{\delta x}$$

$$= \quad \lim_{\delta x\to 0}\cos\left(x+\frac{\delta x}{2}\right)\times\lim_{\delta x\to 0}\frac{\sin\frac{\delta x}{2}}{\frac{\delta x}{2}}$$

$$= \quad \cos(x)\times\lim_{\frac{\delta x}{2}\to 0}\frac{\sin\frac{\delta x}{2}}{\frac{\delta x}{2}}$$

$$= \quad \cos x\times 1=\cos x$$

$$\therefore \quad \frac{d}{dx}(\sin x)=\cos x. \qquad \text{(Ans)}$$

Q. If x = a cos θ and y = a sin³θ then find $\frac{d^2y}{dx^2}$ **[2019w,2d]**

Ans: Given that

$x=a\cos^3\theta$, $y=a\sin^3\theta$

$\frac{dx}{d\theta}=-3a\cos^2\theta,\sin\theta$, $\frac{dy}{d\theta}=3a\sin^2\theta.\cos\theta$

$$\frac{dy}{dx}=\frac{dy}{d\theta}.\frac{d\theta}{dx}=\frac{3a\sin^2\theta.\cos\theta}{-3a\cos^2\theta.\sin\theta}=-\tan\theta$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{d\theta}(-\tan\theta).\frac{d\theta}{dx}=-\sec^2\theta.\frac{d\theta}{dx}$$

$$=\left(-\sec^2\theta.\frac{1}{-3a\cos^2\theta}.\sin\theta\right)=\frac{1}{3a}\sec^2\theta.\cos ec\theta$$

Q. find the points of local maximum and local minimum, if any of the function f(x) = (x-1) (x+2)². Also find maximum and minimum value of f(x). [2016,4b]

Ans: We have

$f(x) = (x - 1)(x + 2)^2$

$f'(x) = (x + 2)^2 \times 1 + (x - 1) \times 2\ (x + 2)$

$= (x^2 = 4x + 4) + 2(x^2 + x - 2)$

$= 3x^2 + 6x$

For maximum or minimum $f'(x) = 0$

$\Rightarrow 3x^2 + 6x = 0$

Or $3x(x + 2) = 0$

$\Rightarrow x = 0, -2$

Again $f^{II}(x) = 6x + 6$

At $x = 0$, $f^{II}(x) = 6(0) + 6 = 6 > 0$

At $x = -2$, $f^{II}(x) = 6(-2) + 6 = -6 < 0$

$\therefore$ f(x) has local minimum value at $x = 0$

Value $f(0) = (0-1)(0 + 2)^2 = -4$

And f(x) has local maximum value at $x = -2$

Local maximum value , $f(-2) = (-2-1)(-2+2)^2$

$= (-3)(0) = 0$

03.c Find the extreme points of the function , y = x + 1/x [2017,5c]

Ans: y = x + 1/x ------------------------(1)

Diff. with respect to x

$$\Rightarrow \frac{y}{dx} = 1 - \frac{1}{x^2}$$

$$\text{let } \frac{dy}{dx} = 0$$

$$\Rightarrow \quad 1 - \frac{1}{x^2 = 0} \qquad \Rightarrow 1 = \frac{1}{x^2}$$

$$\Rightarrow \quad x^2 = 1$$

$$\Rightarrow \quad x = \pm 1$$

$$\text{Again differentiate , } \frac{d^2y}{dx^2} = \frac{2x}{x^4} = \frac{2}{x^3}$$

$$\therefore \quad \frac{d^2y}{dx^2} = \frac{2}{x^3}$$

Now $\left.\frac{d^2y}{dx^2}\right|_{x=1} = \frac{2}{1^3} = 2 > 0$

$\therefore$ $x = 1$ is a minimum point

$\left.\frac{d^2y}{dx^2}\right|_{x=-1} = \frac{2}{(-1)^3} = -2 < 0$

$\therefore x = -1$ is a maximum point.

Hence $x = 1$ & -1 are extreme points

Q. Find the points of local maximum and minimum if any of f(x) = $x^3 - 6x^2 + 9x + 7$. Also find maximum and minimum value of x.

Ans: $f(x) = x^3 - 6x^2 + 9x + 7$ [2015w,4b]

$\therefore f^{I}(x) = 3x^2 - 12x + 9$

For maximum, or minimum $f^{I}(x) = 0$

$\Rightarrow$ $3x^2 - 12x + 9 = 0$

Or $3(x^2 - 4x - 3) = 0$

Or, $x^2 - 4x - 3 = 0$

Or, $x^2 - 3x - x - 3 = 0$

Or, $x(x-3) - 1(x-3)$

Or, $(x-1)(x-3) = 0$

X = 1, 3

Function $f^{II}(x) = 6x - 12$

At x = 1, $f^{II}(x) = 6 \times 1 - 12 = -6 < 0$

At x = 3, $f^{II}(x) = 6 \times 3 - 12 = 6 > 0$

$\therefore$ f(x) has local maximum at x = 1 and

local maximum value $f(1) = 1 - 6 + 9 + 7 = 1$

f(x) has local minimum at x = 3 and

local minimum value $f(3) = 27 - 54 + 27 + 7 = 7$

Q. Differentiate $\sin^{-1}x$ w.r.t x [2015(s),2a]

Ans: let $y = \sin^{-1} x$ and $z = x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \frac{dz}{dx} = 1$$

$$\frac{dy}{dz} = \frac{dy/dx}{dz/dx}$$

$$= \frac{\frac{1}{\sqrt{1-x^2}}}{1} = \frac{1}{\sqrt{1-x^2}}$$

Q. Find the derivative of log(log(Logx)) w.r.t. 'x' [2018s,1-ii]

Ans:

$$\frac{d}{dx} \mathrm{Log}\left[\mathrm{Log}\left(\log x\right)\right]$$

$$= \frac{1}{\mathrm{Log}(\mathrm{Log}x)} \frac{d}{dx}\left[\mathrm{Log}(\mathrm{Log}x)\right]$$

$$= \frac{1}{\mathrm{Log}(\mathrm{Log}\,x)} \frac{1}{\mathrm{Log}\,x} \frac{d}{dx}(\log x)$$

$$= \frac{1}{\mathrm{Log}(\mathrm{Log}\,x)} \frac{1}{\mathrm{Log}\,x} \cdot \frac{1}{x}$$

$$= \frac{1}{x\,\mathrm{Log}\,x\,\mathrm{Log}(\mathrm{Log}\,x)}$$

[2017s,2IV]

Q. Determine the slope of the curve y = tan x at x = $\pi/4$

Ans: y = tan x

$$\text{Slope } \frac{dy}{dx} = \sec^2 x$$

$$\left.\frac{dy}{dx}\right]_{x=\frac{\pi}{4}} = \sec^2 \frac{\pi}{4} = \left(\sqrt{2}\right)^2 = 2$$

Q. Differentiate a^x w.r.t. x^a

Ans:

$$y = a^x$$

$$\frac{dy}{dx} = a^x \log a$$

$$z = x^a$$

$$\Rightarrow \quad \frac{dz}{dx} = a\, x^{a-1}$$

$$\frac{dy}{dz} = \frac{dy}{dx} / \frac{dz}{dx}$$

$$= \frac{a^x \log a}{a\ \ x^{a-1}}$$

$$= \frac{a^{x-1}}{x^{a-1}} \log a$$

[2018S,1vii]

Q. Find the slope of the tangent to the curve y = x^2 at x = – ½

Ans:

$$y = x^2 \Rightarrow \quad \frac{dy}{dx} = 2x$$

$$\Rightarrow \quad \left.\frac{dy}{dx}\right]_{at\ x=\frac{-1}{2}} = 2x\ \frac{-1}{2} = -1$$

Slope of the tangent of the curve is –1

Q. f(x) = x $\tan^{-1}$ x, find f^{l} (√3)

Ans: $f(x) = x\tan^{-1}x$

$$f'(x) = \frac{dx}{dx}\tan^{-1}x + x\frac{d}{dx}\tan^{-1}x$$

$$= \tan^{-1}x + \frac{x}{1+x^2}$$

$$f'\left(\sqrt{3}\right) = \tan^{-1}\sqrt{3} + \frac{\sqrt{3}}{1+\left(\sqrt{3}\right)^2}$$

$$= \frac{\pi}{3} + \frac{\sqrt{3}}{4}$$

Q. Find the derivative of $(\tan x)^{\ln x}$ with respect to 'x' [2019w,4a]

Ans: y = $(\tan x)^{\ln x}$

Taking log, on both sides

⇒ log y = log tan x

⇒ log y = log x log tan x

Differentiate w.r.t. 'x'

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\log\tan x + \log x \times \frac{1}{\tan x}.\sec^2 x$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{\log\tan x}{x} + \log x.\frac{\sec^2 x}{\tan x}$$

$$\Rightarrow \quad \frac{dy}{dx} = y\left[\frac{1}{x}\log\tan x + \log x.\frac{\sec^2 x}{\tan x}\right]$$

$$\Rightarrow \quad (\tan x)^{\ln x}\left[\frac{1}{x}\log\tan x + \log x.\frac{\sec^2 x}{\tan x}\right]$$

Q. If y = $\sin^{-1}$x, then s.t. [2019S,6(N)]

$(1 - x^2)\, y_2 - xy_1 = 0$

Ans: $y = \sin^{-1} x$

$$\Rightarrow y_1 = \frac{1}{\sqrt{1-x^2}} \Rightarrow y_1\sqrt{1-x^2} = 1$$

Diff. w.r.t. 'x'

$$\Rightarrow \quad y_2\sqrt{1-x^2} + y_{1x}\frac{-2x}{2\sqrt{1-x^2}} = 0$$

$$\Rightarrow \quad y_2\sqrt{1-x^2} - \frac{xy_1}{\sqrt{1-x^2}} = 0$$

$$\Rightarrow \quad y_2\left(1-x^2\right) - xy_1 = 0$$

Q. find dy/dx , if $x^y = y^x$ [2017s,3a]

Ans: $x^y = y^x$

Taking log in both sides

$\Rightarrow$ $\log x^y = \log y^x$

$\Rightarrow$ $y \log x = x \log y$

Differentiating w.r.t. 'x'

$$\Rightarrow \quad \frac{dy}{dx}\log x + \frac{1}{x}y = \log y + x\frac{1}{y}\times\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}\log x - \frac{x}{y}\frac{dy}{dx} = \log y - \frac{y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx}\left(\log x - \frac{x}{y}\right) = \frac{x\log y - y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x\log y - y}{x} / \frac{y\log x - x}{y}$$

Q. If y= m $\sin^{-1}$x, then P.T. [2015s,5c]

$(1 - x^2)\, y_2 - xy_1 - m^2y = 0$

Ans: $y = e^{m\sin^{-1}x}$

$$\Rightarrow \quad y_1 = \frac{m}{\sqrt{1-x^2}} \times e^{m\sin^{-1}x}$$

$$\Rightarrow \quad y_1\sqrt{1-x^2} = my \text{..........(1)}$$

Diff. w.r.t. 'x'.

$$\Rightarrow \quad y_2\sqrt{1-x^2} - \frac{2xy_1}{2\sqrt{1-x^2}} = my_1$$

$$\Rightarrow \quad y_2(1-x^2) - xy_1 = m\sqrt{1-x^2}\,y_1$$

$$\Rightarrow \quad \left(1-x^2\right)y_2 - xy_1 = m(my) \quad \left(\because my = y\sqrt{1-x^2}\right)$$

$$\Rightarrow \quad \left(1-x^2\right)y_2 - xy - m^2y = 0$$

Q.
Find $\frac{dy}{dx}$, $\ln\sqrt{x^2+y^2} = \tan^{-1}\left(\frac{y}{x}\right)$ [2016(s)]

Ans:

$\ln\sqrt{x^2+y^2} = \tan^{-1}\left(\frac{y}{x}\right)$

Differentiating w.r.t. 'x' both sides

$$\Rightarrow \frac{1}{\sqrt{x^2+y^2}} \times \frac{1}{2\sqrt{x^2+y^2}} \times \frac{d}{dx}\left(x^2+y^2\right) = \frac{1}{1+\left(\frac{y}{x}\right)^2} \times \frac{d}{dx}\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{1}{2\left(x^2+y^2\right)} \times \left(2x+2y\frac{dy}{dx}\right) = \frac{x^2}{x^2+y^2}\left(\frac{\frac{dy}{dx}.x-y}{x^2}\right)$$

$$\Rightarrow \frac{\left(x+y\frac{dy}{dx}\right)}{\left(x^2+y^2\right)} = \frac{x^2}{\left(x^2+y^2\right)} \frac{x\frac{dy}{dx}-y}{x^2}$$

$$\Rightarrow x+y\frac{dy}{dx} = x\frac{dy}{dx} - y$$

$$\Rightarrow x\frac{dy}{dx} - y\frac{dy}{dx} = x+y$$

$$\Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y}$$

Q. Find $\frac{dy}{dx}$, if $x = 0 + \sin\theta, y = 1 + \cos\theta$, **[2016w,3b]**

at $\theta = \frac{\pi}{4}$

Ans: Here $x = \theta + \sin\theta$

$\therefore \frac{dx}{d\theta} = 1 + \cos\theta$

Again, $y = 1+\cos\theta$, $\frac{dy}{d\theta} = -\sin\theta$

$$\frac{dy}{dx} = \frac{dy}{d\theta}.\frac{d\theta}{dx} = \frac{-\sin\theta}{1+\cos\theta}$$

$$\left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{4}} = \frac{-\sin\left(\frac{\pi}{4}\right)}{1+\cos\left(\frac{\pi}{4}\right)} = \frac{-1/\sqrt{2}}{1+\frac{1}{\sqrt{2}}} = \frac{-1}{1+\sqrt{2}}$$

Q. If $y = \tan^{-1}x$, show that $(1+x^2)\, y_2 + 2xy_1 = 0$ [2019S,2C(N)]

Ans: Given, $y = \tan^{-1} x$, $y_1 = \dfrac{1}{1+x^2}$

$\Rightarrow \quad (1+x^2)\, y_1 = 1$

Again Differentiating w.r.t. x.

$(1 + x^2) \,.\, d/dx\, (y_1) + y_1,\, d/dx\, (1+x^2) = 0$

$\Rightarrow \quad (1+x^2)\, y_2 + y_1 \,.\, 2xy_1 = 0$

$\Rightarrow \quad (1+x^2)\, y_2 + 2xy_1 = 0$

Q. Prove that $z = \sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$ then **[2017,w,6b]**

$$x = \frac{dz}{dx} + y\,\frac{dz}{dy} = \tan z$$

Ans Given $z = \sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$

Let $u = \dfrac{x^2+y^2}{x+y}$

so that $z = \sin^{-1} y$, Here $u = \sin z$

Given, $u = \dfrac{x^2+y^2}{x+y}$, since u is a homogeneous

function of n & y of degree so by Euler's theorm,

$x\dfrac{\partial u}{dx} + y\dfrac{du}{dy} = 1.u = u$

Given $u = \sin z$

$\dfrac{du}{dx} = \cos z.\dfrac{dz}{dx}.\dfrac{du}{dy} = \cos z.\dfrac{dz}{dy}$

$\therefore \quad x\dfrac{du}{dx} + y\dfrac{du}{dy} = u$

$\Rightarrow \quad x\cos z\dfrac{dz}{dx} + y\cos z\dfrac{dz}{dy} = \sin z$

$\Rightarrow \quad \cos z\left(x\dfrac{dz}{dx} + y\dfrac{dz}{dy}\right) = \sin z$

$\Rightarrow \quad \left(x\dfrac{dz}{dx} + y\dfrac{dz}{dy}\right) = \dfrac{\sin z}{\cos z} = \tan z$

Q. If $f(x,y) = \sin^{-1}(x/y)$, then find f_x and f_y [2019w,1d(N)]

Ans: $f(x,y) = \sin^{-1}\left(\frac{x}{y}\right)$

$$f_x = \frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}} \times \frac{\partial}{\partial x}\left(\frac{x}{y}\right)$$

$$= \frac{y}{\sqrt{y^2 - x^2}} \times \frac{1}{y}$$

$$= \frac{1}{\sqrt{y^2 - x^2}}$$

$$f_y = \frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}} \times \frac{\partial}{\partial x}\left(\frac{x}{y}\right)$$

$$= \frac{y}{\sqrt{y^2 - x^2}} \times \frac{-x}{y^2}$$

$$= \frac{-x}{y\sqrt{y^2 - x^2}}$$

Q. Verify Euler's theorem for

Z = y/x

Ans: Z = y/x

By Euler's theorem

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz = 0.z = 0$$

$$n = 0$$

$$\frac{\partial z}{\partial x} = \frac{-y}{x^2}$$

$$\frac{\partial z}{\partial y} = \frac{1}{x}$$

L.H.S

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$$

$$= x \times \frac{-y}{x^2} + y \times \frac{1}{x}$$

$$= \frac{-y}{x} + \frac{y}{x} = 0 = 0.z$$

Q. Find dy/dx if $y^3 + 3x^2y - 2x = 10$ [2014, (s)3a]

Ans:Given data

$$y^3 + 3x^2y - 2x = 10$$

$$\Rightarrow \frac{d}{dx}(y^3) + \frac{d}{dx}(3x^2y) - \frac{d}{dx}(2x) = \frac{d}{dx}(10)$$

$$\Rightarrow 3y^2\frac{dy}{dx} + 3\left[x^2.\frac{d}{dx}(y) + y.\frac{d}{dx}(x)^2\right] - 2\frac{d}{dx}(x) = 0$$

$$\Rightarrow 3y^2\frac{dy}{dx} + 3\left[x^2\frac{dy}{dx} + y.2x\right] - 2 = 0$$

$$\Rightarrow 3y^2\frac{dy}{dx} + 3x^2.\frac{dy}{dx} + 6xy - 2 = 0$$

$$\Rightarrow 3(y^2 + x^2)\frac{dy}{dx} = 2 - 6xy$$

$$\therefore \quad \frac{dy}{dx} = \frac{(2-6xy)}{3(y^2+x^2)}$$

Q,. If z = f(z) show that $x\frac{dz}{dx} + y\frac{dz}{dy} = 0$ **[2014s,3b]**

Ans:Given z = f(y/x)

$$\frac{dz}{dx} = f'\left(\frac{y}{x}\right)\frac{d}{dx}\left(\frac{y}{x}\right) = f'\left(\frac{y}{x}\right)\left(-\frac{y}{x^2}\right)$$

$$\frac{dz}{dy} = f'\left(\frac{y}{x}\right).\frac{d}{dy}\left(\frac{y}{x}\right) = f'\left(\frac{y}{x}\right).\left(\frac{1}{x}\right)$$

$$\therefore x = \frac{dz}{dx} + y\frac{dz}{dy}$$

$$\therefore x = f'\left(\frac{y}{x}\right).\left(\frac{-y}{x^2}\right) + y.\left(\frac{y}{x}\right).\left(\frac{1}{x}\right)$$

$$= \frac{-y}{x}f\left(\frac{y}{x}\right) + \frac{y}{x}f'\left(\frac{y}{x}\right) = 0$$

INTGRATION

CHAPTER-4

Q. **[2015s,4i]**

Evaluate $\int \sec^3 x \tan x \, dx$

$$= \int \sec^2 x \sec \tan x \, dx$$

$$\left[\begin{array}{ll} \text{Let} & \sec x = t \\ \Rightarrow & \sec x \tan dx = dt \end{array}\right.$$

$$= \int t^2 \; dt$$

$$= \frac{t^3}{3} + C \qquad = \qquad = \frac{\sec^3 x}{3} + C$$

Q: Find $\int \frac{e^x}{x}\left(1 + x\ell nx\right)dx$

Ans : $\int \frac{e^x}{x}\left(1 + x\ell nx\right)dx$

$$= \int e^x \left(\frac{1}{x} + \ell nx\right)dx$$

Let $f(x) = \ell nx, f'(x)\frac{1}{x}$

we know that $\int e^x \left\{f(x) + f'(x)\right\}dx$

$$= e^x f(x) + c$$

Q. Integrate $\int \frac{\cos ec^2 x}{1+\cot x} dx$ **[2016,7a]**

$$\left[\begin{array}{ll} \text{let} & 1+\cot x = t \\ \Rightarrow & -\cos ec^2 x = \frac{dt}{dx} \\ \Rightarrow & \cos ec^2 x\, dx = -dt \end{array}\right.$$

$$= \int \frac{-dt}{t}$$

$$= -\ln t + C$$

$$= \ln 1 \_ \cot x | + C$$

Q. Evaluate $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx$

$$= \int e^t dt$$

$$= e^t + C$$

$$= e^{\tan^{-1} x} + C$$

$$\left[\begin{array}{ll} \text{let} & \tan^{-1} x = t \\ \Rightarrow & \frac{dx}{1+x^2} = dt \end{array}\right.$$

Q. Evaluate $\int_0^{\frac{\pi}{2}} \ln \tan x\, dx$ **[2019w,2c(o)]**

Ans: $I = \int_0^{\frac{\pi}{2}} \ln \tan x\, dx$

$\Rightarrow$ $I = \int_0^{\frac{\pi}{2}} \ln \tan\left(\frac{\pi}{2} - x\right) dx$

$\Rightarrow$ $I = \int_0^{\frac{\pi}{2}} \ln \cot x\, dx$

$\Rightarrow$ $I = \int_0^{\frac{\pi}{2}} \ln \frac{1}{\tan x} dx$

$\Rightarrow$ $I = \int_0^{\frac{\pi}{2}} (\ln 1 - \ln \tan x)\, dx$

$\Rightarrow$ $I = \int_0^{\frac{\pi}{2}} (0 - \ln \tan x)\, dx$

$\Rightarrow$ $I = -\int_0^{\frac{\pi}{2}} \ln(\tan x)\, dx$

$\Rightarrow$ $I = -1$

$\Rightarrow$ $2I = 0$

$\Rightarrow$ $O = 0$

$\Rightarrow$ $I = \int_0^{\frac{\pi}{2}} \ln \tan x\, dx = 0$

Q. Integrate ∫log x dx **[2017w,6a]**

Ans: $= \int 1.\log x \, dx$

$= \log x \int 1\,dx - \int \left[\frac{d}{dx} \log x \int 1\,dx \right] dx$

$= \log x.x - \int \frac{1}{x}.x \, dx$

$= x \log x - \int 1 \, dx$

$= x \log x - x + C$

$= x\left(\log x - 1\right) + C$

Q. Integrate $\int_{-1}^{1} |x| \, dx$ **[2019W,1i(O)]**

Ans:

$= \int_{-1}^{0} |x| \, dx + \int_{-1}^{0} |x| \, dx$

$= \int_{-1}^{0} -x \, dx + \int_{0}^{1} x \, dx$

$= \left[-\frac{x^2}{2} \right]_{-1}^{0} + \left[\frac{x^2}{2} \right]_{0}^{1}$

$= 0 + \frac{1}{2} + \frac{1}{2}$

$= 1$

[2019w,1e(n)]

Q. $\int \sqrt{1+\sin 2x} \, dx$

Ans: $= \int \sqrt{\left(\cos x + \sin x\right)^2} \, dx$

$= \int \left(\cos x + \sin x\right) dx$

$= \sin x - \cos x + c$

Q. $\int e^x \left\{\cot x + \text{Log}(\sin x)\right\} dx$

Ans: $= \int e^x \cot x \, dx + \int e^x \,\text{Log}(\sin x) dx$

$= \int e^x \cot x \, dx + \int e^x \,\text{Log}(\sin x) e^x - \int \frac{1}{\sin x} \cos x \, e^x dx$

$= \int e^x \cot x \, dx + e^x \,\text{Log}(\sin x) - \int e^x \cot x \, dx$

$= e^x \,\text{Log}\,(\sin x) + c$

Q. Integrate $\int e^{2x} \sin x\, dx$ [2019w,2g(o)]

Ans: $\int e^{2x} \sin x\, dx = I(say)$

$$= \sin x \int e^{2x}dx - \int \left\{\frac{d}{dx}(\sin x)\int e^{2x}dx\right\}dx$$

$$= \sin x\left(\frac{e^{2x}}{2}\right) - \int \cos x\ \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}}{2}\sin x - \frac{1}{2}\int e^{2x}\cos x\ dx$$

$$= \frac{e^{2x}}{2}\sin x - \frac{1}{2}\left[\cos x\int e^{2x}dx - \int \left\{\frac{d}{dx}(\cos x)\int e^{2x}dx\right\}dx\right]$$

$$= \frac{e^{2x}}{2}\sin x - \frac{1}{2}\left[\cos x\int \left(\frac{e^{2x}}{2}\right) - \int (-\sin x)\left(\frac{e^{2x}}{2}\right)dx\right]$$

$$= \frac{e^{2x}}{2}\sin x - \frac{1}{2}\left[\frac{e^{2x}}{2}\cos x + \frac{1}{2}\int e^{2x}\sin x\, dx\right]$$

$$I = \frac{e^{2x}}{2}\sin x - = \frac{e^{2x}}{4}\cos x - \frac{1}{4}\int e^{2x}\sin x\, dx$$

$$I = \frac{e^{2x}}{2}\sin x - = \frac{e^{2x}}{4}\cos x - \frac{1}{4}I$$

$$\Rightarrow I + \frac{1}{4}I = \frac{e^{2x}}{4}\left[2\sin x - \cos x\right] + C$$

$$\Rightarrow \frac{5}{4}I = \frac{e^{2x}}{4}\left[2\sin x - \cos x\right] + C$$

$$\Rightarrow I = \frac{4}{5}\times\frac{e^{2x}}{4}\left[2\sin x - \cos x\right] + C$$

$$\Rightarrow I = \frac{e^{2x}}{5}\left[2\sin x - \cos x\right] + C$$

Q. Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cot x}$ **[2019S,-2h(N)]**

Ans: $= \int_0^{\frac{\pi}{2}} \frac{dx}{1+\cot x} = I(say)$

$$= \int_0^{\frac{\pi}{2}} \frac{dx}{1+\cot\left(\frac{\pi}{2}-x\right)}$$

$$= \int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x}$$

$$= \int_0^{\frac{\pi}{2}} \frac{dx}{1+\frac{1}{\cot x}}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cot x\ \ dx}{1+\cot x}$$

Q. **Integrate** **[2018s,Q-6]**

$$\int \frac{\cos\theta}{\sqrt{4\sin^2\theta+1}}\,d\theta$$

Ans:

$$\int \frac{\cos\theta\, d\theta}{\sqrt{4\sin^2\theta+1}}$$

Let $\sin\theta = t$

$\cos\theta\, d\theta = dt$

$$= \int \frac{dt}{\sqrt{4t^2+1}}$$

$$= \int \frac{dt}{\sqrt{4\left(t^2+\frac{1}{4}\right)}}$$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{(t)^2+\left(\frac{1}{2}\right)^2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = \log x + \sqrt{x^2+a^2} + c$$

$$= \frac{1}{2}\left[\log\left\{t+\sqrt{t^2+\frac{1}{4}}\right\}\right] + c$$

$$= \frac{1}{2}\left[\log\left\{t+\sqrt{\frac{4t^2+1}{4}}\right\}\right] + c$$

$$= \frac{1}{2}\log\left\{2t+\sqrt{4t^2+1}\right\} + c$$

$$= \frac{1}{2}\log\left\{2\sin\theta+\sqrt{4\sin^2\theta+1}\right\} + c$$

Q. Evaluate $\int \log(1+x^2)\,dx$ [2017w,Q-5]

Ans: $\int \log(1+x^2)dx$

$\int 1.\log(1+x^2)\,dx$

$$= \log\left(1+x^2\right)\int 1\,dx - \int \left[\frac{d}{dx}\left\{\log\left(1+x^2\right)\int 1\,dx\right\}\right]dx$$

$$= x\log\left(1+x^2\right) - \int \frac{1}{1+x^2}\times 2x \times x\,dx$$

$$= x\log\left(1+x^2\right) - \int \frac{2x^2}{1+x^2}dx$$

$$= x\log\left(1+x^2\right) - 2\int \frac{1+x^2-1}{1+x^2}dx$$

$$= x\log\left(1+x^2\right) - 2\int \left(1-\frac{1}{1+x^2}\right)dx$$

$$= x\log\left(1+x^2\right) - 2\int 1dx + 2\int \frac{1}{1+x^2}dx$$

$$= x\log\left(1+x^2\right) - 2x + 2\tan x + c$$

Q. Integrate $\int x\tan^{-1}x\,dx$ [2014W,6b]

Ans:

$$= \tan^{-1}x\int x\,dx - \int \left\{\frac{d}{dx}\tan^{-1}x\int x\,dx\right\}dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \int \frac{1}{1+x^2}\quad\frac{x^2}{2}dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{x^2}{x^2+1}dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{x^2+1-1}{x^2+1}dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \left(1-\frac{1}{x^2+1}\right)dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left(x-\tan^{-1}x\right) + c$$

$$= \frac{x^2}{2}\tan^{-1}x + \frac{\tan^{-1}x}{2} - \frac{x}{2} + c$$

$$= \frac{1}{2}\left(x^2\tan^{-1}x + \tan^{-1}x - x\right) + c$$

Q. [2019w,6a(o)]

Evaluate $\quad I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$ ---------- (i)

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)} dx$$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx$ ------------- (ii)

Adding equation(i) & (ii) we get

$$\Rightarrow \quad I + I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$$

$$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} 1.dx$$

$$\Rightarrow \quad 2I = \left[x\right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow \quad I = \frac{1}{2}\left[\frac{\pi}{2} - 0\right]$$

$$\Rightarrow \quad I = \frac{\pi}{4}$$

$$\Rightarrow \quad \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx = \frac{\pi}{4}$$

Q. Find the area bounded by the curve $x^2 + y^2 = 4$ [2019w,5(o)]

Ans: Equation of the circle

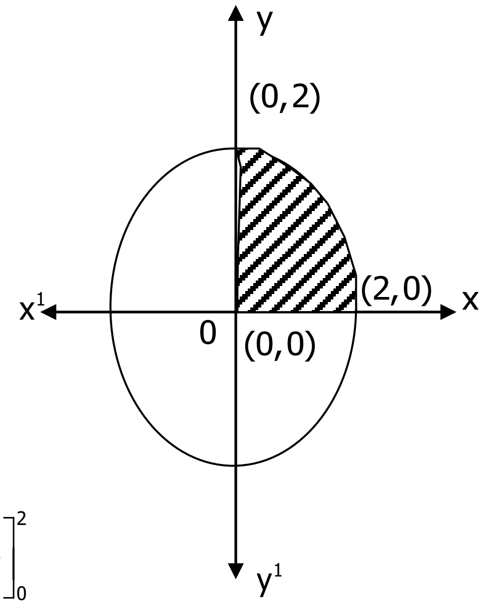
$X^2 + y^2 = 4$

$\Rightarrow \quad y^2 = 4 - x^2$

$\Rightarrow \quad y = \sqrt{4-x^2} \qquad (0 < x < 2)$

Radius of the circle = 2

Circle symmetric to both axis



$$= 4 \times \int_0^2 \sqrt{4 - x^2} dx$$

$$= 4 \times \int_0^2 \sqrt{2^2 - x^2} dx$$

$$= 4 \times \left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

Area of the circle $= 4\left[0 + \frac{4}{2}\sin^{-1}\frac{1}{1} - 0 - 2\sin^{-1} 0\right]$

$$= 4\left(2.\frac{\pi}{2} - 0\right)$$

$$= 4\pi$$

Q. Integrate I = ∫ e^{3x} sin 2x dx **[2018W,Q-5]**

$$\Rightarrow I = \sin 2x\int e^{3x}dx - \int \left[\frac{d}{dx}\sin 2x\int e^{3x}dx\right]dx$$

$$\Rightarrow I = \frac{e^{3x}}{3}\sin 2x - \int 2\cos 2x\frac{e^{3x}}{3}dx$$

$$\Rightarrow I = \frac{e^{3x}}{3}\sin 2x - \frac{2}{3}\int e^{3x}\cos x\, dx$$

$$\Rightarrow I = \frac{e^{3x}}{3}\sin 2x - \frac{2}{3}\left[\cos 2x\frac{e^{3x}}{3} - \int(-2)\sin 2x\frac{e^{3x}}{3}dx\right]$$

$$\Rightarrow I = \frac{e^{3x}}{3}\sin 2x - \frac{2}{9}e^{3x}\cos 2x - \frac{4}{9}\int e^{x}\sin 2x\, dx$$

$$\Rightarrow I = \frac{e^{3x}}{3}\sin 2x - \frac{2}{9}e^{3x}\cos 2x - \frac{4I}{9}$$

$$\Rightarrow I + \frac{4I}{9} = \frac{e^{3x}}{3}\sin 2x - \frac{2}{9}e^{3x}\cos 2x$$

$$\Rightarrow \frac{13I}{9} = \frac{e^{3x}}{9}(3\sin 2x - 2\cos 2x)$$

$$\Rightarrow = \frac{e^{3x}}{13}(3\sin 2x - 2\cos 2x) + C$$

DIFFERENTIAL EQUATION (CHAPTER - 5)

Q. Determine the order and degree of the differential equation

[2016,7a]

$$2\frac{d^2y}{dx^2} + 3\sqrt{1-\left(\frac{dy}{dx}\right)^2 - y} = 0$$

$$\Rightarrow \quad 2\frac{d^2y}{dx^2} = -3\sqrt{1-\left(\frac{dy}{dx}\right)^2 - y}$$

$$\Rightarrow \quad 4\left(\frac{d^2y}{dx^2}\right)^2 = 9\left[1-\left(\frac{dy}{dx}\right)^2 - y\right]$$

order = 2

Degree = 2

Q. Find the order and degree of differential equation **[2015s,6a]**

$$\frac{d^3y}{dx^3} = \left[5 + \left(\frac{dy}{dx}\right)^3\right]^{2/3}$$

$$\Rightarrow \quad \left(\frac{d^3y}{dx^3}\right)^3 = \left[5 + \left(\frac{dy}{dx}\right)^3\right]^2$$

order $= 3$

Degree $= 3$

Q. Solve $e^x\sqrt{1-y^2}dx + \frac{y}{x}dy = 0$ **[2016,7c]**

$$\Rightarrow \quad e^x\sqrt{1-y^2}dx = -\frac{y}{x}dy$$

$$\Rightarrow \quad x\,e^x\,dx = \frac{-y}{\sqrt{1-y^2}}dy$$

Integrate both sides

$$\Rightarrow \quad \int x\,e^x dx = -\int \frac{y}{\sqrt{1-y^2}}dy$$

$$\left[\begin{array}{l} \text{let } 1-y^2 = t \\ \Rightarrow -2y = \frac{dt}{dy} \\ \Rightarrow -ydy = \frac{dt}{2} \end{array}\right.$$

$$\Rightarrow \quad xe^x - \int 1.e^x dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$\Rightarrow \quad xe^x - e^x = \sqrt{t} + C$$

$$\Rightarrow \quad (x-1)\,e^x - \sqrt{1-y^2} = C$$

Q. Solve $\frac{dy}{dt} = \frac{e^{\sin^{-1}t}\sin^{-1}t}{\sqrt{1-t^2}}$ **[2018s,Q-6(O)]**

$$\Rightarrow \quad dy = \frac{e^{\sin^{-1}t}\sin^{-1}t}{\sqrt{1-t^2}}dt$$

Integrating both sides

$$\Rightarrow \int dy = \int \frac{e^{\sin^{-1}t}\sin^{-1}t}{\sqrt{1-t^2}}dt$$

$$\left[\begin{array}{l} \text{Let } \sin^{-1}t = Z \\ \Rightarrow \frac{1}{\sqrt{1-t^2}}dt = dz \end{array}\right.$$

$$\Rightarrow \quad \int dy = \int e^Z\, Z\, dZ$$

$$\Rightarrow \quad y = z\;e^z - \int 1.e^z\;dz$$

$$\Rightarrow \quad y = z\;e^z - e^z + c$$

$$\Rightarrow \quad y = (z-1)\,e^z + c$$

$$\Rightarrow \quad y = \left(\sin^{-1}t - 1\right)e^{\sin^{-1}t} + c$$

Q. Solve dy/dx–y=e^x **[2015,7a(s)]**

Ans: $P=-1\ Q=e^{x}$

$1.f=e^{\int p\,dx}$

$=e^{\int -1dx}=e^{-x}$

$ye^{-x}=\int e^{x}\ \ e^{-x}\ dx$

$\Rightarrow \quad y\,e^{-x}=\int dx$

$\Rightarrow \quad y\,e^{-x}=x+c$

$\Rightarrow \quad y\,e^{-x}-x=c$

Q. $y^2dx+(xe^y+2y)\,dy=0$ **[2015,6b]**

Ans: this is an exact equation

Here M = e^y and N = $ex^y + 2y$

Now $\frac{\partial M}{\partial y}=e^{y}$ and $\frac{\partial N}{\partial x}=\frac{\partial}{\partial x}\left(xe^{y}+2y\right)$

$=e^{y}\times\frac{\partial}{\partial x}(x)+\frac{\partial}{\partial x}(2y)$

$=e^{y}\times 1+0=e^{y}$

Thus the condition of exactness is satisfied and the given equation is exact.

Its solution is given by : $\int mdx+\int Ndy=c$

(yas cons tan t) (only those term which do not ontain x)

$\text{i.e.}\int e^{y}\ dx+\int\ 2ydy=c$

$\Rightarrow \quad e^{y}\int 1\ dx+2\int ydy=c$

$\Rightarrow \quad xe^{y}+2\left(\frac{y^{2}}{2}\right)=c$

$\Rightarrow \quad xe^{y}+y^{2}=c$

Q. **Sove e^x tan y dx + (1+e^x) sec^{2}y dy = 0** **[2015w,7a(o)]**

Ans: $\Rightarrow \quad e^x \tan dx = -\left(1+e^x\right)\sec^2 y\, dy$

$\Rightarrow \quad \frac{e^x}{1+e^x}dx = -\frac{\sec^2 y}{\tan y}dy$

$\Rightarrow \quad \int \frac{e^x}{1+e^x}dx = -\frac{\sec^2 y}{\tan y}dy$

$\left[\begin{array}{l}\text{Let } 1+e^x = t \\ \Rightarrow e^x dx = dt\end{array}\right.$

$\left[\begin{array}{l}\text{Let } \tan y = z \\ \Rightarrow \sec^2 y\, dy = dz\end{array}\right.$

$\Rightarrow \quad \int \frac{dt}{t} = -\int \frac{dz}{z}$

$\Rightarrow$ Int = – l n z + l n c

$\Rightarrow$ l n t + l n z = l n c

$\Rightarrow$ l n t z = l n c

$\Rightarrow$ t z = c

$\Rightarrow$ (1 + e^x) tan y = c

Q; Solve $\left(1+x^2\right)\frac{dy}{dx}+2xy = x^3$

$\Rightarrow \quad \frac{dy}{dx}+\frac{2xy}{1+x^2} = \frac{x^3}{1+x^2}$ **[2019w,2d(o)]**

$P = \frac{2x}{1+x^2} \qquad Q = \frac{x^3}{1+x^2}$

$\text{I.f} = e^{\int \frac{2x}{1+x^2}dx}$

$= e^{\ln 1+x^2}$

$= 1+x^2$

$y\left(1+x^2\right) = \int \frac{x^3}{1+x^2}\left(1+x^2\right)dx$

$\Rightarrow \quad y\left(1+x^2\right) = \int x^3 \; dx$

$\Rightarrow \quad y\left(1+x^2\right) = \frac{x^4}{4}+c$

$\Rightarrow \quad y = \frac{x^4}{4\left(1+x^2\right)}+\frac{c}{1+x^2}$

6a. Find the order and degree of the differential equation (2017s,7a)

$$\left(\frac{dy}{dx}\right)^2 + y^3 = \frac{d^3y}{dx^3}$$

Ans : $\text{order} = 3$

$\text{degree} = 1$

5c. Solve the differential equation (2017s,7c)

$$\left(1 - x^2\right)\frac{dy}{dx} - xy = 1$$

Ans : $\left(1 - x^2\right)\frac{dy}{dx} - xy = 1$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{xy}{1 - x^2} = \frac{1}{1 - x^2}$$

This is a linear diff. equation in y.

Here $p(x) = \frac{-x}{1 - x^2}$, $Q(x) = \frac{1}{1 - x^2}$

$$I.F = e^{\int pdx} = e^{\int \frac{-x\,dx}{1-x^2}}$$

$$\begin{bmatrix} \text{Let} & 1 - x^2 = t \\ \Rightarrow & -2x\,dx = dt \\ \Rightarrow & -xed = \frac{dt}{2} \end{bmatrix}$$

$$= \quad e^{\int \frac{dt}{2t}} = e^{\frac{1}{2}\ell n\, t} = e^{\ell n t^{\frac{1}{2}}}$$

$$= \quad t^{\frac{1}{2}} = \left(1 - x^2\right)^{\frac{1}{2}}$$

$$y \times I.F = \int I.F \times Q(n)\, dx$$

$$\Rightarrow \quad y\left(1-x^2\right)^{\frac{1}{2}} = \int \left(1-x^2\right)^{\frac{1}{2}} \times \frac{1}{1-x^2}\, dx$$

$$\Rightarrow \quad y\left(1-x^2\right)^{\frac{1}{2}} = \int \frac{dx}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad y\left(1-x^2\right)^{\frac{1}{2}} = \sin^{-1} x + c$$

$$\Rightarrow \quad y = \frac{\sin^{-1} x}{\left(1-x^2\right)^{\frac{1}{2}}} + \frac{c}{\left(1-x^2\right)^{\frac{1}{2}}} \qquad \text{(Ans)}$$