BALASORE SCHOOL OF ENGINEERING,BALASORE

SUBJECT-ANALYSIS OF STRUCTURE

(CET-401)

BRANCH-CIVIL ENGG.

SEMESTER-4TH

ER.DEEPALI BARIK

CHAPTER – 1

Short questions (2 marks)

Q1. Define statically determinate and statically indeterminate structure

2015, 2(a)

Ans: Statically determinate Structure : - Conditions of equilibrium are sufficient to fully analyse the structure, i.e. called as statically determinate structure.

Statically Indeterminate structure:- Conditions of equilibrium are insufficient analyse the structure fully, i.e. called as statically indeterminate structure.

Q.2. What is a deficient frame ? Write down the equation 2013(s), 1(a), 2016(s)

Ans: A deficient frame is an imperfect frame. In which the number of members are less than (2J-3)

Equation n < (2J – 3)

Where, n = number of members

J = number of joints.

Q.3. What do you mean by a portal frame 2016, 7(a)

Ans: The portal frame is an example for a statically indeterminate structure. This frame can be analysed by moment distribution method, slope deflection method etc.

Q.4. How a truss differs from a beam? 2015, 1(a) , 2013, 1(b)

Ans: The only difference between truss and beam is that a

→ Truss transmits the force only in the axial direction.

→ Beam transmits force both in axial & vertical direction.

Q.5. Explain how method of Joint differs over the method of section in plane truss ? 2014, 7(a)

Ans: Only two unknown forces can be found by method of joints where as three unknown forces can be found by method of sections.

Q.6. Write down the equation for the maximum deflection of a simple supported beam of Span 'l' which a central point load (w) . 2017 (s) 2(a)

Ans:- Max^m Deflection , $Y_c = \frac{wl^3}{48EI}$

Where W=point load

L= span of beam

E= Modulus of elasticity

I = moment of inertia

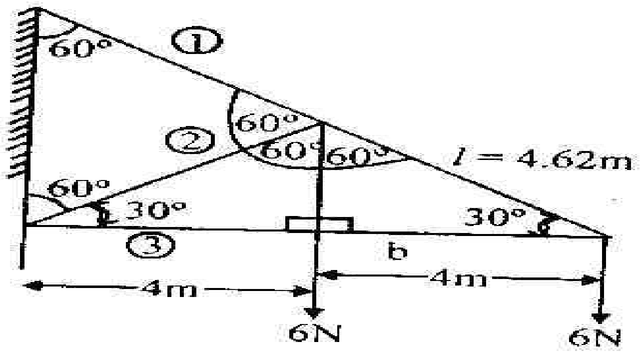
Q.7. Write down the statement of second the orem of moment area method. 2017 3(a)

Ans: Mohr's theorem-II

It states 'the intercept taken on a vertical reference line of tangents at any two points on an elastic curve, is equal to the moment of the B.M. diagram between these points about the reference line divided by EI.'

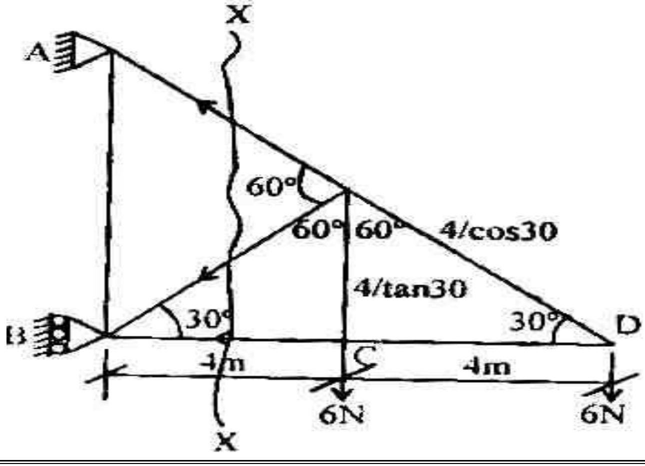
5 MARKS

Q.1. By using method of section, find out the axial forces along member1, 2 and 3 as shown in the figure. 2014, 7(c)



Ans: Considering RHS of the section X – X

$\Sigma M_E = 0 \Rightarrow F_{CB} \times EC + 6 \times 4 = 0$



or $F_{CB} = \dfrac{(-)6 \times 4}{4/\tan 30} = (-)6/\tan 30 = 3.46$ kN

(compress)

$\sum M_B = 0 \Rightarrow F_{EA} \cos 30 \times \dfrac{4}{\cos 30} = 6 \times 4 + 6 \times 8$

or $F_{EA} = \dfrac{72}{4} = 18$ kN (Tension)

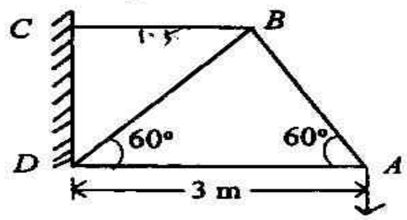
$\sum V = 0 \Rightarrow F_{EA} \sin 30 - F_{EB} \sin 30 - 12 = 0$

or $18 \times 0.5 - F_{EB} \times 0.5 - 12 = 0$

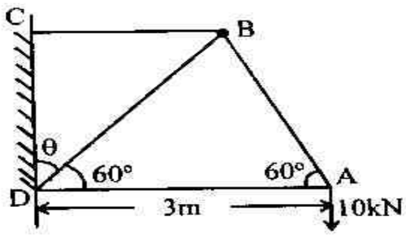
or $9 - 12 = E_{EB} \times 0.5$

or $F_{EB} = \dfrac{-3}{0.5} = (-)6$kN(compress)

Q.2. Find the forces in various member of the truss as shown in figure and tabulate the results 2015 1(b)



Ans:



Let Q be the inclination of the member BD with vertical.

$\theta = 30^o$, DA = 3 m

Joint A, resolving vertically

$P_{AB} \sin 60^o = 10000$ N = 10 kN

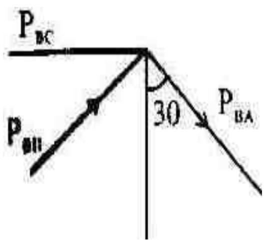
$P_{AB} = 11.55$ kN (Tensile)

Resolving horizontally,

$P_{AD} + P_{AB} \cos 60^o = 0.$

$A_{AD} = -5.78$ kN (compressive)

Joint B,



Resolving vertically,

$P_{BA} \cos 30^o + P_{BP} \cos 30^o = 0.$

$P_{BA} \cos 30^o = - P_{BD} \cos 30^o.$

$P_{BA} = - P_{BD}$ {We know $P_{BA} = P_{AB}$)

$P_{BD} = -11.55$ kN (compressive)

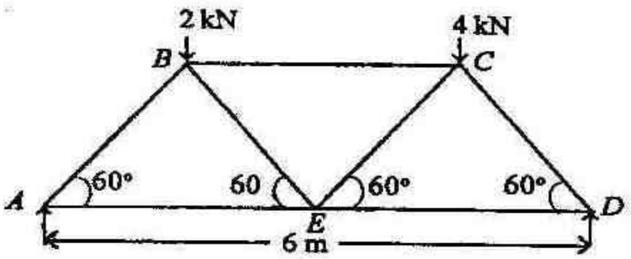
Resolving horizontally,

$P_{BC} = P_{BD} \sin 30^o + P_{BA} \sin 3o^o$

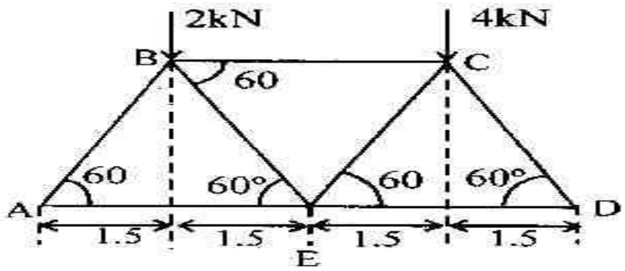
$P_{BC} = -11.55 \times ½ + 11.55 \times ½ = 0.$

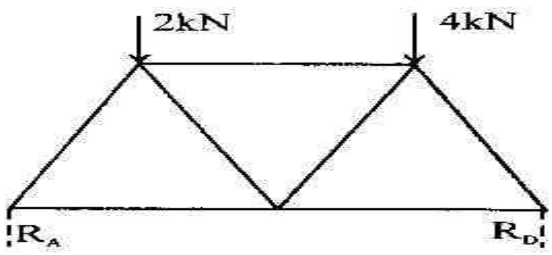
Member	Tensile	compressive
AB	11.55	
AD		5.78
BC	0	0
BD		11.55

Q.3. Find out forces in all the members with their nature as tensile or compressive as shown in figure below. 2015 (s) 3(c)



Ans:





Support reactions,

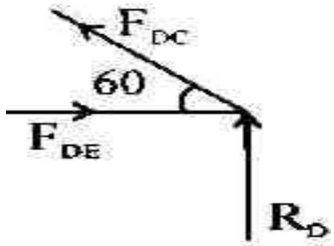
$R_D \times 6 = 4 \times 4.5 + 2 \times 1.5$

$R_D \times 6 = 21$

$R_D = \frac{21}{6} = 3.5\text{kN}$

Member forces $R_A = (2+4) - 3.5 = 2.5\text{kN}$

Jo int D.



$\sum V = 0$

$F_{DC} \sin 60 + R_D = 0$

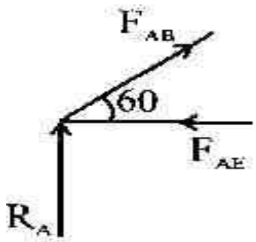
$F_{DC} = \frac{-R_D}{\sin 60} = -4.04kN(\text{compression})$

$\sum H = 0$

$F_{DE} + F_{DC} \cos 60 = 0$

$F_{DE} = 2kN(\text{Tension})$

Joint A



$\sum V = 0$

$R_A + F_{AB} \sin 60 = 0$

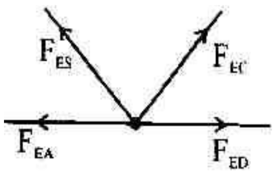
$F_{AB} = \frac{-R_A}{\sin 60} = -2.89kN(\text{compression})$

$\sum H = 0$

$F_{AE} + F_{AB} \cos 60^\circ = 0$

$F_{AE} = 1.45kN(\text{Tension})$

Joint E



$\sum V = 0$

$F_{EB} = \sin 60 + F_{EC} \sin 60 = 0$

$F_{EB} = -F_{EC}$

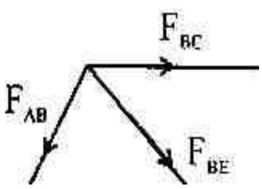
$\sum H = 0$

$F_{EA} + F_{EB} \cos 60^\circ - F_{CD} - F_{EC} \cos 60^\circ = 0$

$F_{EA} - F_{ED} - F_{EC} \cos 60^\circ = 0.$

$F_{BC} = -0.55 (\text{comprssion})$

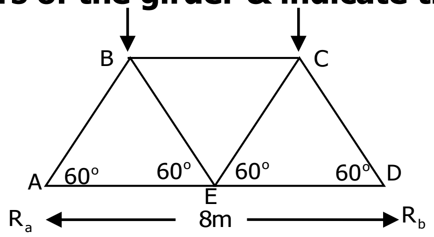
Joint B

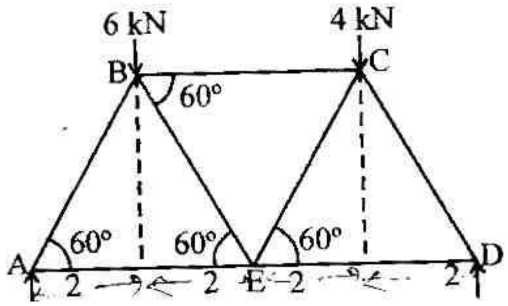


$$\sum H = 0$$
$$H_{BC} + F_{AB} \cos 60 + F_{BE} \cos 60 = 0$$
$$F_{BC} = 1.17(\text{Tensile})$$

7 MARKS

Q.1. The girder is loaded at 'B' & 'C' as shown in the fig. find the forces in all members of the girder & indicate the nature of forces.2014, 1(c)





Support reactions

$R_D \times 8 = 6 \times 2 + 4 \times 6 = 36$

or $\quad R_D = \dfrac{36}{8} = 4.5 \text{ kN}$

$\quad R_A = (6 + 4) - 4.5 = 5.5 \text{ kN}.$

Member forces

Joint D

$\sum V = 0 \Rightarrow F_{CD} \sin 60° + R_D = 0$

or $\quad F_{CD} \times 0.866 = -4.5$

or $\quad F_{CD} = \dfrac{-4.5}{0.866} = (-)5.2 \text{ kN (compression)}$

$\sum H \Rightarrow F_{CD} \cos 60° + F_{ED} = 0$

or $\quad -5.2 \times 0.5 + F_{ED} = 0$

or $\quad F_{ED} = 2.6 \text{ kN(Tension)}$

Joint A

$\sum V = 0 \Rightarrow F_{AB} \sin 60° + R_A = 0$

or $\quad F_{AB} \times 0.866 + 5.5 = 0$

or $\quad F_{AB} = \dfrac{-5.5}{0.866} = (-)6.35 \text{ kN (comprssion)}$

$\sum H \Rightarrow F_{AB} \cos 60° + F_{AE} = 0$

or $\quad -6.35 \times 0.5 + F_{AB} = 0$

or $\quad F_{AE} = 3.18 \text{ kN (Tension)}$

$\sum V = 0 \Rightarrow F_{EB} \sin 60^\circ + F_{EC} \sin 60^0 = 0$

or $\quad F_{EB} + F_{EC} = 0$

or $\quad F_{EB} = -F_{EC} = 0$

$\sum H = 0 \Rightarrow F_{AB} - F_{ED} + F_{FB} \cos 60^\circ - F_{EC} \cos 60^0 = 0$

or $\quad 3.18 - 2.6 - 2F_{EC} \cos 60^0 = 0$

or $\quad F_{EC} = \dfrac{0.58}{2 \times 0.5} = 0.58 \text{ kN (Tension)}$

Joint B

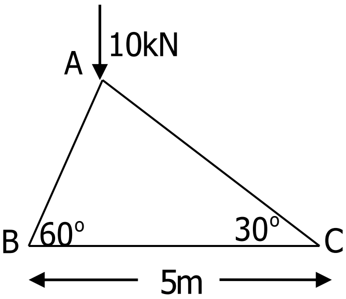
$\sum H = 0$

$F_{BC} + F_{BE} \cos 60^\circ - F_{AB} \cos 60^\circ = 0$

$F_{BC} + (-0.58) \times 0.5 + 6.35 \times 0.5 = 0$

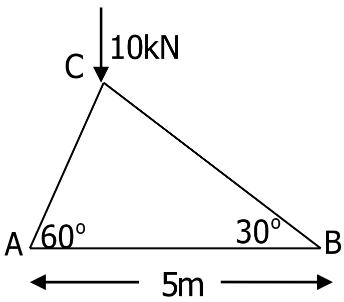
$F_{BC} = -2.89 \text{kN (compression)}$

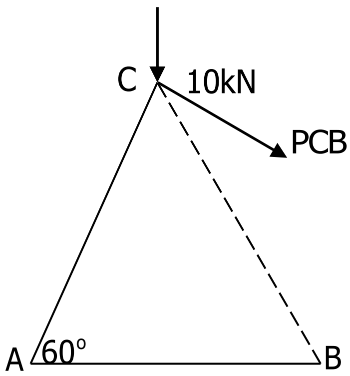
Q.2. Find the force in the member BC shown in following fig. Find out the force on members using method of section. 2016(s) 1(c)



Ans: Section 1 -1 as shown in fig.

To find the forces in BC. Consider the left part of section 1-1 & take moments about A





$\sum V = 0$

$10 \times AC \cos 60^\circ + P_{CB} \times AC = 0$

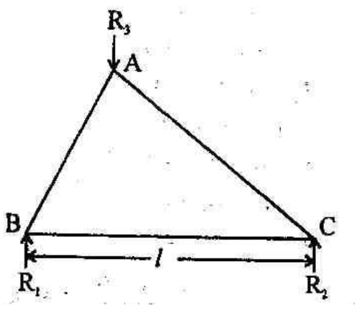
$P_{CB} = \dfrac{-10 AC \cos 60^\circ}{AC}$

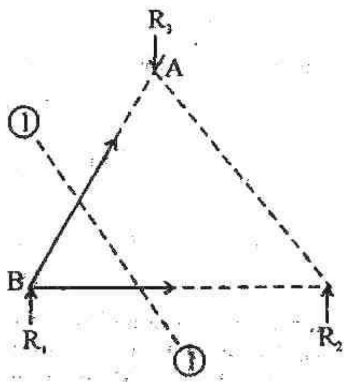
$P_{CB} = -10 \times \dfrac{1}{2} = -5 \text{kN}$

$P_{CB} = 5 \text{ kN (compression)}$

Q.3. Find out the force in members using method of section 2013, (3)

Ans:





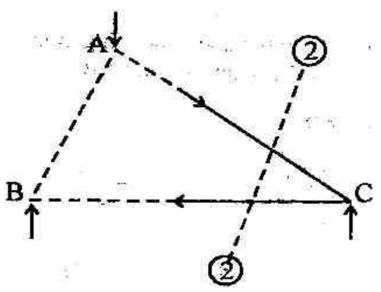
Taking moments of the forces acting in the left part of the truss only about the joint C and equating the same, $P_{AB} \times l \sin 60^o = R_1 \times l$

$$P_{AB} = \frac{R_1 \times l}{l \sin 60^o} = \frac{R_1 l}{l \sin 60^o} = 1.16 R_1 KN(\text{compression})$$

Now taking moments of the forces acting in the left part of the truss only about the joint A and equating the same.

$$P_{BC} \times \frac{l}{4} \tan 60^o = R_1 \times \frac{l}{4}$$

$$P_{BC} = \frac{R_1 \times \frac{l}{4}}{\frac{l}{4} \tan 60^o} = \frac{R_1}{\tan 60^o} = 0.57\ R_1 KN(\text{Tension})$$



Taking moments of the force acting in the right part of the truss only about the joint B and equating the same.

$$P_{AC} \times l \sin 30^o = R_2 \times l$$

$$P_{AC} = \frac{R_1 l}{l \sin 30^o} = \frac{R_2}{\sin 30^o} = 2\ R_2 KN(\text{compression})$$

Now taking moments of the forces acting in the left part of the truss only about the joint A and equating the same

$$P_{BC} \times \frac{3l}{4} \tan 30^o = R_2 \times \frac{3l}{4}$$

$$P_{BC} = \frac{R_2 \times \frac{3l}{4}}{\frac{3l}{4} \tan 30^o} = \frac{R_2}{\tan 30^o} = 1.57\ R_2 KN(\text{Tension})$$

CHAPTER:2

2 MARKS

Q.2. State the slope and deflection equation for a member by (i) moment area method (ii) Double integration method. 2014, (2a)

Ans:i) Moment area method

$$\text{Slope equation } \theta = \int_{x1}^{x2} \frac{Md_x}{EI}$$

$$\text{Deflection equation } z = \int_{x1}^{x} \frac{M_x d_x}{EI}$$

ii) Double integration method

$$\text{Slope equation} = EI.\frac{dy}{dx} = \int Md_x = f'(x)$$

$$\text{Deflection equation } EI.Y = \int f'(x)dx = f(x)$$

Q.3. Write down the relationship between the slope deflection and radius of curvature. 2016, 3(a) , 2013 , 1(e)

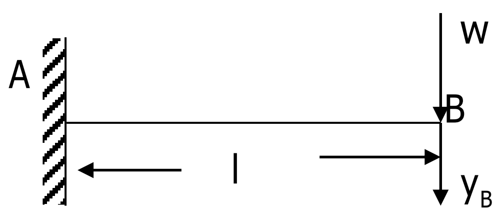
Ans: The relation between the slope, deflection and radius of curvature is

$$M = EI \times \frac{d^2y}{dx^2}$$

Q.4. Write down the equation of deflection at free end of a cantilever beam of span 'l' subjected to a point load 'w' at free end. 2016, 4(a) 2013 1(c)

Ans: Equation of deflection,

$$Y_B = \frac{Wl^3}{3EI}$$



Where w = point load

L = span of beam

E = Modulus of elasticity

I = M.I. of the beam

Q.5. Write down the statement of second theorem of moment area method.

2016, 5(a), 2013 1(g)

Ans: Moment area method

Mohr's theorem-II It states the intercept taken on a vertical reference line of tangents at any two points on an elastic curve, is equal to the moment of the B.M. diagram between these points about the reference line divided by EI.

Q Write down the equation for the maximum deflection of a simple supported beam of Span 'l' which a central point load (w) . 2017 (s) 2(a)

Ans:- Max^3 Deflection , $Y_C = \frac{wl^3}{48EI}$

Where W=point load

L= span of beam

E= Modulus of elasticity

I = moment of inertia

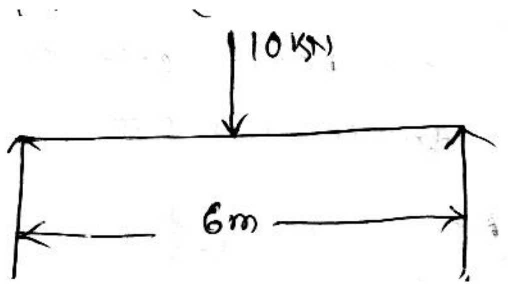
Q. Write down the statement of second the orem of moment area method. 2017 3(a)

Ans: Mohr's theorem-II

It states 'the intercept taken on a vertical reference line of tangents at any two points on an elastic curve, is equal to the moment of the B.M. diagram between these points about the reference line divided by EI.

Q) Calculate the max^n slope & deflection in case of a S/s beam of span 6m subjected to a point load 10kn at the middle of the span EI constant?2017 5 (b)

ANS.



In this case as the load is symmetrically placed the deflection will be max^m at the mid span & slope shall be max^m at the ends.

Qmax = $\frac{A}{EI}$

Area of shaded.

$\Delta = \frac{1}{2} \times \frac{wl}{4} \times \frac{l}{2}$

$= \frac{1}{2} \times \frac{10\times 6}{4} \times \frac{6}{2}$

$= \frac{360}{16}$ =22.s

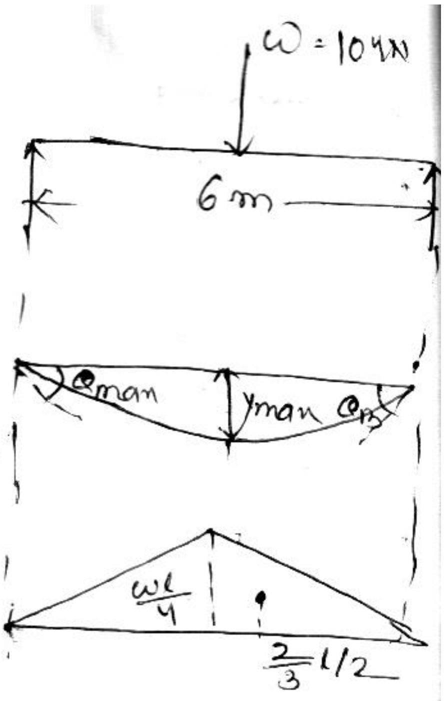
Qmax = 22.5
EI

Ymax = $\frac{Ax}{EI}$

$\frac{Wl^2}{16EL} \cdot \frac{l}{3}$

$= \frac{Wl^3}{48EI}$

$= \frac{10x(6)^3}{48xEI} = \frac{45}{EI}$



5 MARKS

Q.1. A cantilever beam of 3 m long carried a point load of 40 kN at its free end. Find the slope and deflection of the cantilever under the load. Take flexural rigidity for the cantilever beam as 25 × 10^{12} N-mm^2 2014, 1(b)

Ans: Given data

Length, L = 3m = 3000 mm

Point load, W = 40 kN = 40,000 N.

Flexural rigidity EI = 25 × 1012 N – mm^2

(i) Slope at the free end is $i_B = \frac{WL^2}{2EI}$

$$= \frac{40,000 \times 3000^2}{2 \times 25 \times 10^{12}} = \frac{3.6 \times 10^{11}}{5 \times 10^{13}}$$
$$= 0.0072 \text{ rad.}$$

(ii) DEflection at the free end is

$$y_B = \frac{WL^3}{3EI} = \frac{40,000 \times 3000^3}{3 \times 25 \times 10^{12}} = 14.4\text{mm}$$

Q.2. A simply supported beam of span 8 meters is subjected to a point load of 60 kN at the centre. Determine the maximum deflection at the centre by using moment area method. Take EI of the beam section as 10 × 10^2 N-mm^2.

2014, 2(b)

Ans: Given data

Span (l) 8 m = 8 × 10^3 mm

Point load (w) = 60 kN = 60 × 10^3 N

Flexural rigidity (EI) = 10 × 10^{12} N-mm^2

Maximum deflection of the beam at its centre

$$y_c = \frac{WL^3}{48EI} = \frac{\left(60 \times 10^3\right) \times \left(8 \times 10^3\right)^3}{48 \times 10 \times 10^{12}} = 64\text{mm}$$

Q. A cantilever beam 120 mm wide and 160 mm deep is 2 m long. Determine the slope and deflection at the free end of the beam. When it carries a point load of 30 kN at its free end. Take E for the cantilever beam as 200 GPa.
2014, 6(b)

Ans: Width, b = 120 mm
Depth, d = 160 mm
Span, 1 = 2 m = 2000 mm
Point load , w = 30 kN = 30 × 10^3 N
E = 200 GPa = 200 × 10^3 N/m^3
Slope at the free end
We know that moment of inertia of the beam section

$$I = \frac{bd^3}{12} = \frac{120 \times 160^3}{12} = 40.96 \times 10^6 mm^4$$

and slope at the free end

$$i_B = \frac{wl^2}{2EI} = \frac{30 \times 10^3 \times 2000^2}{2 \times 200 \times 10^3 \times 40.96 \times 10^6} = 0.0073 rad$$

Deflection of the free end

$$y_B = \frac{wl^3}{3EI} = \frac{30 \times 10^3 \times 2000^3}{3 \times 200 \times 10^3 \times 40.96 \times 10^6} = 9.76 mm$$

Q.4. Derive the slope and deflection of simply supported beam with a central point load 'W' with span length 'L' by double integration method 2014,7(b)

Ans: Simply supported beam with a central load

$$M_x = \frac{W}{2}x$$

$$\therefore EI\frac{d^2y}{dx^2} = -\frac{W}{2}x$$

$$\therefore EI\frac{dy}{dx} = -\frac{Wx^2}{4} + C_1$$

$$\text{At } x = \frac{1}{2}, \frac{dy}{dx} = 0$$

(As beam isloaded symmetrically)

$$\theta = -\frac{wl^2}{16} + C_1$$

$$\therefore C_1 = \frac{wl^2}{16}$$

$$\therefore EI\frac{dy}{dx} = -\frac{wx^2}{4} + \frac{wl^2}{16}$$

$$Ely = -\frac{wx^3}{12} + \frac{wl^2}{16}x + C_2$$

At $x = 0, y = 0$

$$\therefore C_2 = 0$$

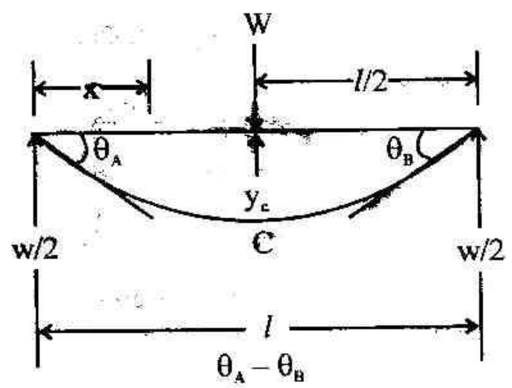
$$\therefore Ely = -\frac{wx^3}{12} + \frac{wl^2}{16}x$$

$x = 0$

$$\text{slope } \theta_A = \frac{wl^2}{16EI}$$

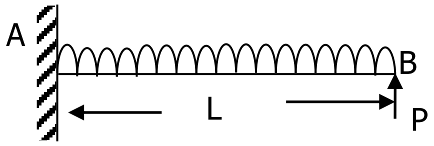
At $x = l/2$

$$\text{Defletion } yc = \frac{wl^2}{48EI}$$

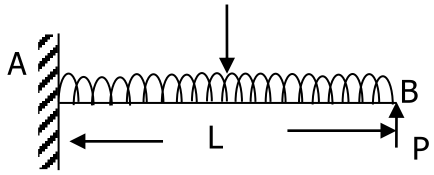


Q.5. Find the reaction at the propped end of the cantilever as shown in figure.

2015 , 3(b)



Ans:



Downward deflection of the cantilever due to fore 'P'

$$y_a = \frac{Pl^3}{3EI}$$

Since both the deflection are equal.

$$\frac{Pl^3}{3EI} = \frac{Wl^4}{8EI}$$

$$P = \frac{3Wl}{8} = \frac{3W}{8}$$

Shear force dia

Shear force at B.

$$F_B = \frac{-3wl}{8}$$

$$F_A = \frac{5wl}{8}$$

Let M be the pt at a distance x from B, where shear force charges sign.

$$\frac{x}{1-x} = \frac{3}{5} \Rightarrow 5x = 31 - 3x$$

$$x = \frac{31}{8}$$

This the shear force is zero at a distance 31/8 from B

Bending moment dia.

$$M_B = 0$$

$$M_A = \frac{3wl}{8}.1 - \frac{wl^2}{2} = \frac{wl^2}{8}$$

Bending moment will be maximum at M., Where shear force changes sign

$$M_m = \frac{3wl}{8}\left(\frac{31}{8}\right) - \frac{w}{2}\left(\frac{31}{8}\right)^2 = \frac{9wl^2}{128}$$

Bending moment at any section x at a distance x from the propped end B.

$$M_m = \frac{3wl}{8}.x - \frac{wa^2}{2}$$

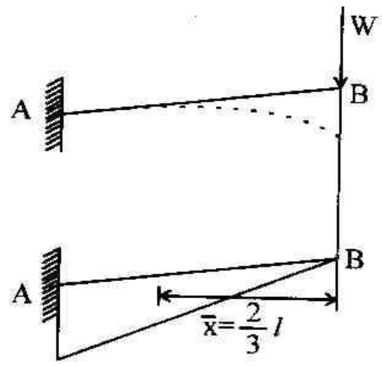
Find out the pt. of contraflexure, let up equate this bending moment to zero

$$\frac{3wl}{8}.x - \frac{wx^2}{2} = 0$$

$$\therefore x = \frac{31}{4}$$

Q. 6. Derive expression for slope and deflection of a cantilever carrying point load at its free end using moment area method 2015 7(b)

Ans:



Let l be the length of cantilever. Let the point load W applied at B.

Let the slope at B be θ_b.

$$\Rightarrow \theta_b \frac{\text{Area of bending moment diagram between A \& B}}{EI}$$

Area of bending moment diagram

$$\frac{1}{2}\ell, wl = \frac{w\ell^2}{2}$$

$$\Rightarrow \theta_b = \frac{w\ell^2}{2EI}$$

Let the deflection of B with respect to A be y_b.

$$\Rightarrow y_b = \frac{Ax}{EI}$$

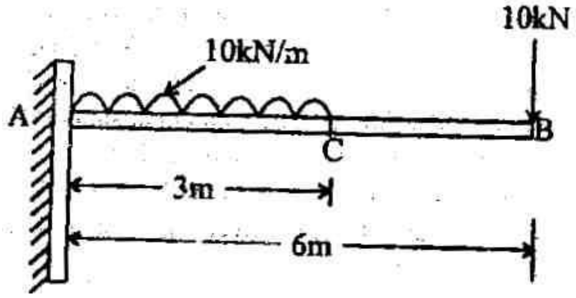
$$x = \frac{2}{3}\ell$$

$$y_b = \frac{w\ell^3}{2EI}.\frac{2}{3}\ell = \frac{w\ell^3}{3EI}$$

7 MARKS

Q.7. Calculate maximum slope and deflection in case of a cantilever of span 6m subjected to a point load of 10 kN at free end and UDL of 10 kN/m over half span from fixed end. Take EI constant. 2013 2(b), 2016 2(c)

Ans:



Given load at free end (W) = 10KN = 10 × 10^3 N. length = AB(l) = 6m = 6 × 10^3 mm, Udl AC(w) = 10 kN/m = 10 N/mm lngth AC(l_1) = 3m = 3 × 10^3 mm. E = 200 GPa = 200 × 10^3 N/mm^2, I = 100 × 10^6 mm^4.

Slope at the free end :

$$i_B = \left[\frac{Wl^2}{2EI}\right] + \left[\left(\frac{Wl^2}{6EI}\right) - \left(\frac{w(l-l)^3}{6EI}\right)\right]$$

$$= \left(\frac{(10\times10^3)\times(6\times10^3)^2}{2\times(200\times10^3)(100\times10^6)}\right) +$$

$$\left[\left(\frac{10(6\times10^3)^3}{6\times(200\times10^3)(100\times10^6)}\right) - \left(\frac{10\times\left[(6\times10^3)-(3\times10^3)\right]^3}{6(200\times10^3)(100\times10^6)}\right)\right]$$

$$= 0.009 + (0.018 + 1.75) = 1.777$$

Deflection at the free end :

$$y_B = \left[\frac{wl^3}{3EI}\right] + \left[\frac{wl^4}{8EI}\right] -$$

$$\left[\left(\frac{W(l-l_1)^4}{8EI}\right) + \left(\frac{W(l-l_1)^3 l}{6EI}\right)\right]$$

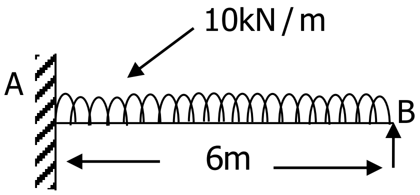
$$= \left(\frac{(10\times10^3)\times(6\times10^3)^3}{3(200\times10^3)(100\times10^6)}\right) + \left(\frac{(10)\times(6\times10^3)^4}{8(200\times10^3)(100\times10^6)}\right)$$

$$-\left[\left(\frac{10(6\times10^3)-(3\times10^3)^4}{8(200\times10^3)(100\times10^6)}\right) + \frac{10\left[(6\times10^3)-(3\times10^3)\right]^3(6\times10^3)}{6(200\times10^3)(100\times10^3)}\right]$$

$$= 36 + 81 - (75.93 + 94500)$$

$$= 117 - (94575.93) = -94458.93$$

Q.8. Find out the prop. Reactional of a cantilever beam of span 6m subjected to an UDL 10 kN/m² over whole span. The beam is propped at free end. 2013,2(c)



Given : length = 6m, load 10 KN / m

We know that proportion reaction.

$$P = \frac{3wl}{8} = \frac{3 \times 10 \times 6}{8} = \frac{180}{8} = 22.5KN$$

Q.9. Write down the assumption in slope deflection method. (2013)

Ans: Consider a bam AV subjected to a bending moment. As a result of loading let the beam deflect from ACB to ADB into a circular arc, as shown in figure below.

Let L = Length of the beam AB.

M = Bending moment.

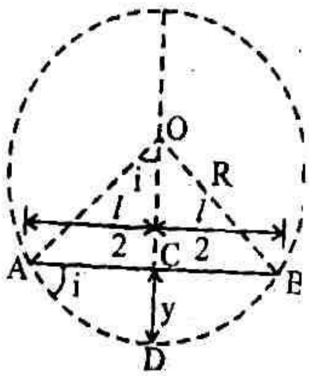
R = Radius of curvature of the bent up beam

I = Moment of inertia of the beam section.

E =Modulus of elasticity of beam material.

Y = Deflection of the beam.

P = Slope of the beam.



From the geometry of a circle, we know that

$$AC \times CB = EC \times CD$$

$$\frac{l}{2} \times \frac{l}{2} = (2R - y) \times y$$

$$\frac{l^2}{4} = 2\,Ry - y^2 = 2Ry$$

$$y = \frac{l^2}{8R}$$

we know that $\frac{M}{I} = \frac{E}{R}$

$$R = \frac{EI}{M}$$

Now substituting this value of R in equation (i)

$$y = \frac{l^2}{8 \times \frac{EI}{M}} = \frac{ml^2}{8EI}$$

From the geometry of the figure, we find that the slope of the beam I at A or B is also equal to angle AOC.

$$\sin i = \frac{AC}{OA} = \frac{L}{2R}$$

Since the angle i is very small, therefore sin i may be taken equal to

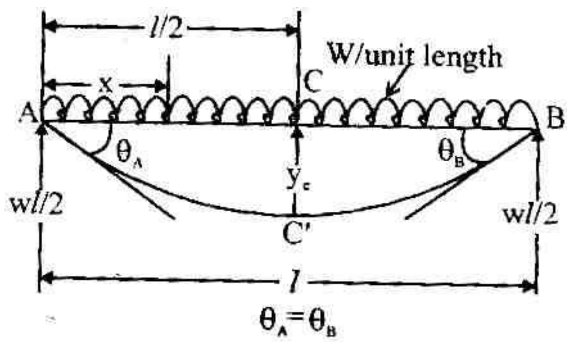
$$i = \frac{l}{2R} \text{radians}$$

Again substituting the value of R in equation (ii)

$$i = \frac{l}{2R} = \frac{l}{2 \times \frac{EI}{M}} = \frac{ml}{2EI} \text{radians}$$

Q.1. Derive an expression for the slope and deflection of a simply supported beam subjected to a UDL of w/unit length. 2014 2(c)

Ans: Simply supported beam with uniformly distributed load



$$M_x = \frac{wl}{2}x - \frac{wx^2}{2}$$

$$EI\frac{d^2y}{dx^2} = -\left(\frac{wl}{2}x - \frac{wx^2}{2}\right) = -\frac{wlx}{2} + \frac{wx^2}{2}$$

$$EI\frac{dy}{dx} = \frac{-wlx^2}{4} + \frac{wx^3}{6} + C_1$$

$$x = \frac{1}{2}, \frac{dy}{dx} = 0 \text{ As beam is loaded symmetrically}$$

$$0 = -\frac{wl^3}{16} + \frac{wl^3}{48} + C_1$$

$$C_1 = \frac{wl^3}{24}$$

$$EI\frac{dy}{dx} = \frac{-wlx^2}{4} + \frac{wx^3}{6} + \frac{wl^3}{24}$$

Q.2. Derive the slope and deflection of a cantilever beam with a point load at its free end by double integration method. 2014 6(c)

Ans: Deflection curve

Cantilever with a load at the free end

$$M_x = -W(1-x)$$

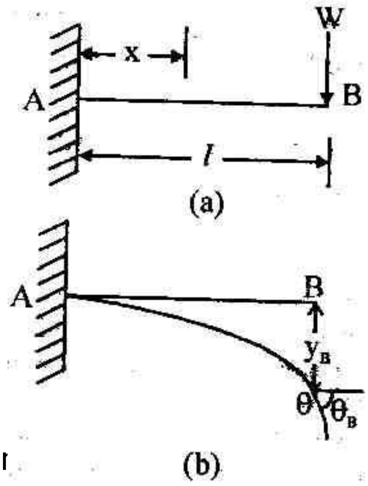
$$\therefore EI\frac{d^2y}{dx^2} = -[-W(1-x)] = Wl - Wx$$

$$\therefore EI\frac{dy}{dx} = Wlx - \frac{Wx^2}{2} + C_1 \text{...........(a)}$$

$$EIy = Wlx - \frac{Wlx^2}{2} - \frac{Wx^2}{6} + C_1x + C_2 \text{...............(b)}$$

at $x = 0$

$$\frac{dy}{dx} = 0 \quad \text{and} \quad y = 0$$



Substituting i

Substituting in b, $C_2 = 0$

$$\therefore EI\frac{dy}{dx} = Wlx - \frac{Wx^2}{2}$$

$$\text{and} \quad EIy = \frac{Wlx^2}{2} - \frac{Wx^3}{6}$$

at $x = 1$

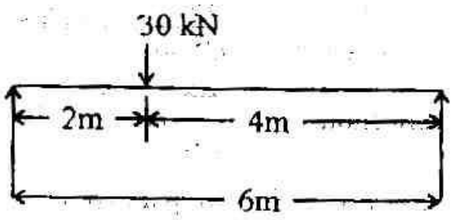
$$\text{Slope } \theta_B = \frac{1}{EI}\left(Wl^2 - \frac{Wl^2}{2}\right) = \frac{Wl^2}{2EI}$$

$$\text{Deflection, } y_B = \frac{1}{EI}\left(\frac{Wl^3}{2} - \frac{Wl^3}{6}\right)$$

$$= \frac{Wl^3}{3EI}$$

Q.3. Derive the expression for maximum slope and maximum deflection in case of a simply supported beam of span 6m subjected to an electrically placed point load 30 kN at distance 2m and 4m respectively from supports. Take EI constant. 2013 (4)

Ans:



Given Span l = 6m = 6×10^3 mm

Point load (wt) = 30 kN = 30×10^3 N

A = 2m = 2×10^3 mm, b = 4mm = 4×10^3 mm

EI = 26×10^{12} N-mm^2

Slope at A

$$i_A = \frac{wb}{6EIL}(l^2 - b^2)$$

$$= \frac{(30\times10^3)\times(4\times10^3)}{6\times(26\times10^{12})(6\times10^3)}\times\left[(6\times10^3)^2-\left(4\times10^3\right)^2\right]$$

$$= \frac{12}{6\times(26\times10^8)\times6}\times\left[(36\times10^6)-(16\times10^6)\right]$$

$$= \frac{12}{36\times(26\times10^8)}(20\times10^6)$$

$$= \frac{12\times2}{36\times(260)} = 0.0025 \text{ rad}$$

Slope at B

$$i_B = \frac{wa}{6EIL}(l^2 - a^2)$$

$$= \frac{(30\times10^3)\times(2\times10^3)}{6\times(26\times10^{12})(6\times10^3)}\times$$

$$\left[(6\times10^3)^2-(2\times10^3)^2\right]$$

$$= \frac{6}{6\times(26\times10^8)\times6}\times\left[(36\times10^6)-(4\times10^6)\right]$$

$$\frac{6}{36\times(26\times10^8)}\times(32\times10^6)$$

$$= \frac{6 \times 2}{36 \times 26 \times 10^2} = \frac{192}{93600} = 0.00205$$

$$\text{Deflection } \gamma_C = \frac{wab}{6EIL}(l^2 - a^2 - b^2)$$

$$= \frac{(30 \times 10^3) \times (2 \times 10^3)(4 \times 10^3}{6 \times (26 \times 10^{12})(6 \times 10^3)} \times \left[(6 \times 10^3)^2 - (2 \times 10^3)^2 - (4 \times 10^3)\right]$$

$$= \frac{3 \times 2 \times 4}{6 \times (26 \times 10^5) \times 6} \times \left[(36 \times 10^6) - (4 \times 10^6) - (16 \times 10^6)\right]$$

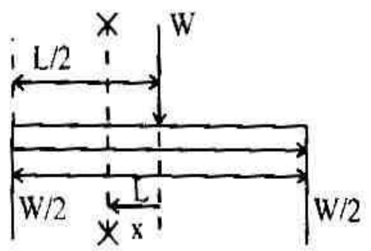
$$= \frac{24}{36 \times (26 \times 10^5)} \times \left[(32 \times 10^6) - (20 \times 10^6)\right]$$

$$= \frac{24}{36 \times (26 \times 10^5)} \times (16 \times 10^6)$$

$$= \frac{24 \times 160}{36 \times 26} = \frac{3840}{936} = 4.102\text{mm}$$

Q.4. Derive an expression for slope and deflection of a simply supported beam subjected to a central point load. 2015 1(c)

Ans: Let us consider a simply supported beam subjected to a central point load.



In order to attain a single expression for bending moment which will apply across the complete beam in this case it is convenient to take the origin for x at the centre, then :

$$M_{xx} = EI\frac{d^2y}{dx^2} = \frac{w}{2}\left(\frac{L}{2} - x\right) = \frac{WL}{4} - \frac{Wx}{2}$$

$$\Rightarrow EI\frac{dy}{dx} = \frac{WL}{4}x - \frac{Wx^2}{4} + A$$

{Integrating both side)

$$\Rightarrow EIy = \frac{WLx^2}{8} - \frac{Wx^2}{12} + Ax + B$$

{Again by int egrating}

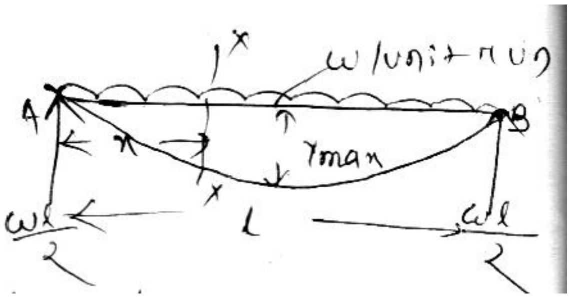
$$\text{At} \quad x = 0, \frac{dy}{dx} = 0, \Rightarrow A = 0$$

$$x = \frac{L}{2} my0, \Rightarrow 0 = \frac{WL^3}{32} = -\frac{WL^3}{48}$$

$$\Rightarrow B = \frac{WL^3}{96} - \frac{WL^3}{32} = -\frac{WL^3}{48}$$

$$y = \frac{1}{EI} = \left[\frac{WLx^2}{8} - \frac{Wx^3}{12} - \frac{WL^3}{48}\right.$$

Q.Find slope and deflection for a s/s beam with a UDL over the span by double integration method 7(c) 2017



Consider a secn x-x at a distance x from the end A

$$Mx = \frac{wl}{2} X\ x - w\,X\ x\,X\ \frac{x}{2}$$

$$= EI\,\frac{d^2Y}{dx^2} = \frac{wlx}{2} - \frac{wx^2}{2}$$

$$\text{int}\,egrating\ \ we\ \ get$$

$$EI = \frac{dy}{dx} = \frac{wl}{2}\frac{x^2}{2} - \frac{w}{2}\frac{x^3}{3} + c1$$

$$i.e.,, x = \frac{1}{2,} \frac{dy}{dx} = 0$$

$$0 = \frac{wl3}{16} - \frac{wl3}{48} + C1$$

$$C1 = -\frac{-wl3}{16} + \frac{wl3}{48} = \frac{3wl3 + wl3}{48}$$

$$C1 = \frac{-wl3}{24}$$

$$EI \frac{dy}{dx} = wl \frac{x2}{4} - \frac{wx3}{6} - \frac{wl3}{24}$$

Slope at "A"

Putting X=0, we $\frac{Wl3}{24}$

θA = $\frac{-wl3}{24EI}$

Integrating the slope eq1 we get

$$EUY = \frac{WL}{4} \frac{x^3}{3} \frac{-w}{6} \frac{x^4}{4} - \frac{wl^3}{24} x + c2$$

$$when\ x = 0,\ y = 0,\ C2 = 0$$

$$EIy = \frac{wlx3}{12} - \frac{wx4}{24} - \frac{wl3x}{24}\ \ Defl^n en^n$$

$$at\ mid\ span\ x = \frac{l}{2}$$

$$we\ getEI\ y\max = \frac{Wl\ x l^3}{12\ x\ 8} - \frac{wl4}{24\ x16} - \frac{wl3\ x\ l}{24\ x\ 2}$$

$$Y\ \max = \frac{-5wl4}{384EI}$$

$$Downward\ \ defl^n\ \ Yman = \frac{5wl^4}{384EI}$$

CHAPTER:3

2 MARKS

Q.1. State any two advantages to fixed beam. 2015 , 3(a)

Ans: i. The fixed beam is subjected to a lesser maximum bending moment than the simply supported beam carrying the same, load.

ii. For the same loading maximum deflection a fixed beam is less than that if the simply supported beam.

Q.2. Write down the expression for fixed and moment for a fixed beam of span 'l' subjected to a point load 'w' at distance 'a' and 'b' from both ends

2013 (s) (New) 1(f)

Ans: The expressions are

$$Ma = \frac{-wab^2}{l^2}$$

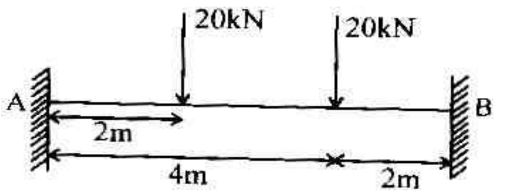
$$Mb = \frac{-wa^2b}{l^2}$$

5 MARKS

Q.1. A fixed beam AB of span 6 m is subjected to two point load of 20 kN and 30 kN at a distance of 2m and 4 m from A. Calculate fixing moment at A and B.

2015 2(b)

Ans:



$$\text{Fixing moment at A} = \sum \frac{Wab^2}{\ell^2}$$

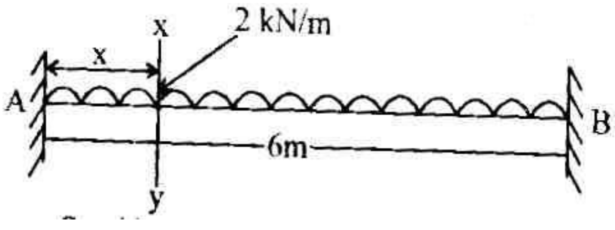
$$= \frac{20\times2\times4^2}{6^2} + \frac{30\times2\times2^2}{6^2} = 31.12\text{kN}-\text{m}.$$

$$\text{Fixing moment at B} = \sum \frac{Wa^2b}{\ell^2}$$

$$= \frac{20\times2^2\times4}{6^2} + \frac{30\times4^2\times2}{6^2} = 35.56\text{kN}-\text{m}.$$

Q.2. A fixed beam AB of span 6m is subjected to a UDL of 2 kN/m. Determine the S.F. & B.M.D., its diagram 2014, 3(b)

Ans: $R_A = R_B = \frac{WL}{2} = \frac{2\times6}{2} = 6\text{kN}$



Consider any section 'X' at a distance x from the left end 'A'. The shear force at the section is :

$$f_x = R_A - w.x = 6 - 2.x.$$

$$\text{At A, } x = 0\text{, hence } F_A = R_A - \frac{wx}{2}$$

$$= 6 - \frac{2\times6}{2} = 6\text{kN}(+)$$

$$\text{At B, } x = 6\text{, hence } F_B = +R_A - w.x$$

$$= 6 - 2\times6 = -6\text{kN}.$$

$$\text{At C, } x = \frac{L}{2} = \frac{6}{2} = 3$$

$$\text{Hence } f_c = +R_A - w.x$$

$$6 - 2\times 3 = 0.$$

$$= 6\times 6 - \frac{2}{2}\times 6^2 = 0$$

$$\text{At, C, } x = \frac{\ell}{2}, \text{hence } M_C = \frac{w\ell}{2}.\frac{L}{2} - \frac{w}{2}.\left(\frac{L}{2}\right)^2$$

$$= 6\times\frac{6}{2} - \frac{2}{2}\times\left(\frac{6}{2}\right)^2 = 18 - 9 = 9 \text{ kN} - \text{m}(+)$$

Fixed end BM at A and B

$$= (-)\frac{Wl^2}{12} = \frac{-2\times 6^2}{12} = -6\text{kNm}$$

Simply supported bending moment at the section 'X' at a distance 'x' from left end 'A' is given by

$$M_x = +R_A - w.x.\frac{x}{2}$$

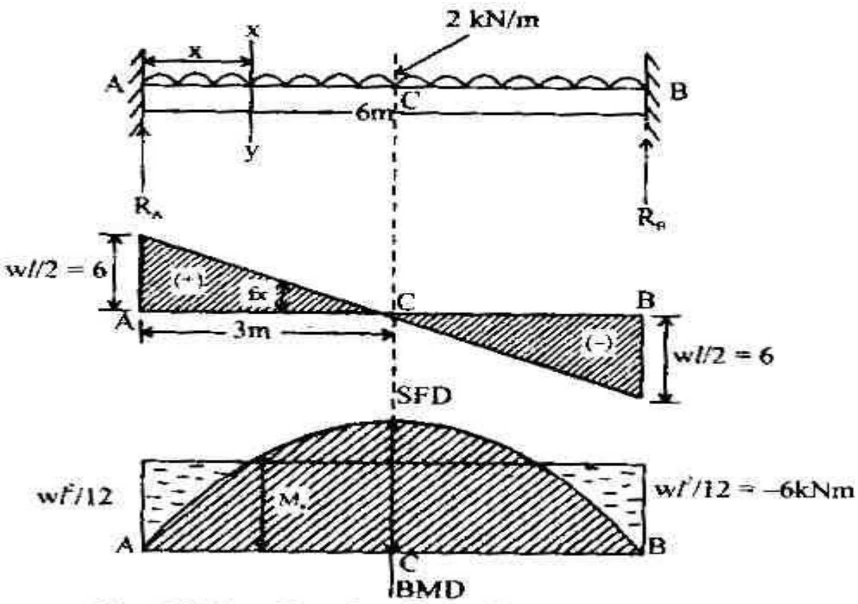
$$= 6x - \frac{2x^2}{2} = 6x - x^2$$

The value of B.M. at different points are:

$$\text{At A, } x = 0, \text{ hence } M_A = \frac{w\ell}{2}\times 0 - \frac{w.0}{2} = 0$$

$$\text{At B, } x = L = 6$$

$$\text{Hence } M_B = \frac{w\ell}{2}.L - \frac{w}{2}L^2$$

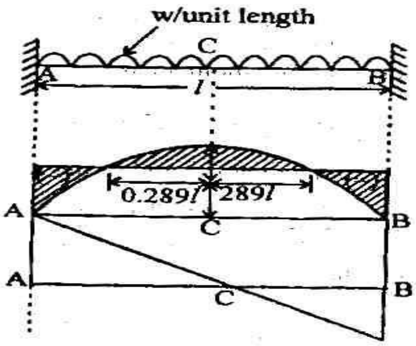


Net BM at C = 9 – 6 = 3 kNm (+)

7 MARKS

Q.1. A fixed beam is subjected to an UDL over whole span. Derive the expression for fixed end moments. 2013, 2(d) 2016, 4(c)

Ans:



Let m_A = Fixing moment at A and

M_B = Fixing moment at B

Since the beam is symmetrical therefore m_A and m_B will also be equal.

Now equating the areas of the two diagrams.

$$m_A l = -\frac{2}{3} l . \frac{wl^2}{8} = -\frac{wl^3}{12}$$

$$m_A = -\frac{wl^2}{12}$$

$$\text{Similarly, } m_B = -\frac{wl^2}{12}$$

We know that maximum positive bending moment at the centre of the beam = $wl^2/8$.

Net positive bending moment at the centre of the beam

$$= \frac{wl^2}{8} - \frac{wl^2}{12} = \frac{wl^2}{24}$$

Shear force diagram, Let R_A = Reaction at A, and R_B = Reaction at B.

Equating the clockwise moments and anticlockwise moments about A.

$$R_B \times l + m_0 = m_B + w \times l \times \frac{l}{2}$$

$$R_B = \frac{wl^2}{8}$$

$$\text{Similarly, } R_A = \frac{wl}{2}$$

Deflection of the beam,

We know that bending moment at any section X, at a distance x from A,

$$M_x = \mu_x - \mu_x' = \left(\frac{wl}{2}x - \frac{wx^2}{2}\right) - \frac{wl^2}{12}$$

$$EI\frac{d^2y}{dx^2} = \left(\frac{wlx}{2} - \frac{wx^2}{2}\right) - \frac{wl^2}{12} \text{..................(i)}$$

Integrating the above equation

$$EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^2x}{12} + C_1$$

Where C_1 is the first constant of integration. We know that when x = 0, the dy/dx = 0

Therefore $C_1 = 0$

$$EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^2x}{12} \text{................(ii)}$$

Integrating the equation (ii) once again

$$EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} \_ C_2$$

Where C_2 is the second constant of integration, We know that when x = 0, then y = 0

Therefore $C_2 = 0$

$$EI.y = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^2x^2}{24} \text{.............(iii)}$$

We know that the maximum deflection occurs at the centre of the beam. Therefore substituting x = l/2 in the above equation.

$$EI.y_c = \frac{wl}{12}\left(\frac{l}{2}\right)^3 - \frac{w}{24}\left(\frac{l}{2}\right)^4 - \frac{wl^2}{24}\left(\frac{l}{2}\right)^2$$

$$= \frac{wl^4}{96} - \frac{wl^4}{384} - \frac{wl^4}{96} = -\frac{wl^4}{384}$$

$$y_c = -\frac{wl^4}{384EI}$$

Point of contraflexures.

The points of contrflexures may be found out by equation (i) to zero.

$$\left(\frac{wlx}{2}-\frac{wx^2}{2}\right)-\frac{wl^2}{12}=0$$

$$lx-x^2-\frac{l^2}{6}=0$$

$$x^2-lx+\frac{l^2}{6}=0$$

Solving this quadratic equation for x,

$$x=\frac{l\pm\sqrt{l^2-\frac{4l^2}{6}}}{2}$$

$$=\frac{l}{2}\pm\frac{l}{2\sqrt{3}}$$

$$=0.5l\pm0.289l=0.789\text{ l and }0.211\text{ l.}$$

CHAPTER:4

2 MARKS

Q.1. Explain theorem of three moment, 2014, 5(a), 2015, 5(a)

Ans: Theorem of three moments: If a beam has a support, the end ones being fixed, then the same number of equations required to determine the support moments may be obtained from the consecutive pairs of spans.

Theorem of three moments states that

$$M_A\left(\frac{l_1}{I_1}\right)+2M_B\left(\frac{l_1}{I_1}+\frac{l_2}{I_1}\right)+M_c\left(\frac{l_2}{I_2}\right)$$

$$=-6\left[\frac{A_1x_1}{I_1l_1}+\frac{A_2x_2}{I_2l_2}\right]$$

Where M_A, M_B, M_C = Support moments at A, B and C.

A_1, A_2 = Area of B.M. diagram for the given loading (s/s)

X_1,x_2 = Distance of centroid of areas A_1 and A_2 from A and C respectively.

l_1,l_2 = Span AB and BC respectively

I_1, I_2 = MI. of span AB and BC respectively.

Q.2. Write down the expression for three moment equation with usual meaning 2013, 2(i)

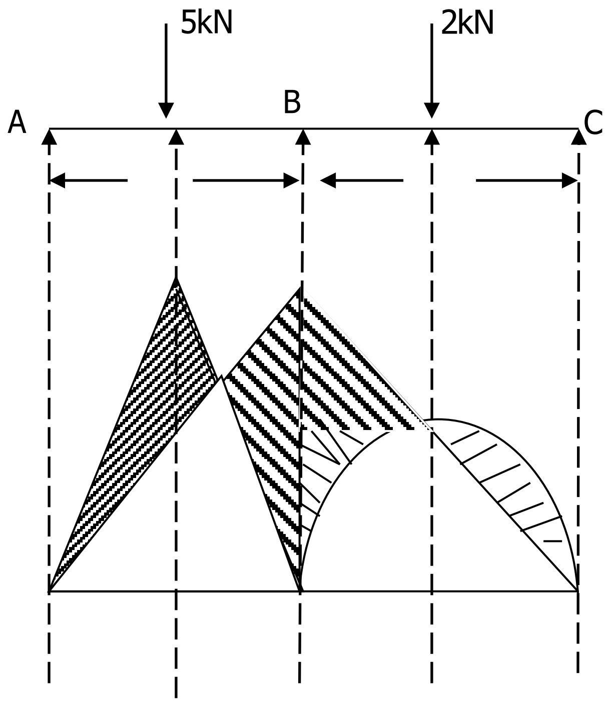
Ans: Three moment equation:-

$$M_A l_1 + 2M_B(l_1 + l_2) + M_0 l_2 = -\left(\frac{6a_1x_1}{l_1} + \frac{6a_2x_2}{l_2}\right)$$

6 MARKS

Q.1. A continuous beam is simply supported over two spans, such that AB = 6m and BC = 4m. It carries uniformly distributed load of 2 kN/m over span BC and a point load of 5 kN at the centre of span AB. Determine the support moment over B by applying theorem of three moments. 2014,2016

Ans:



$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = \frac{6a_1x_1}{l_2} + \frac{6a_2x_2}{l_2}$$

$$M_D = \frac{wl}{4} = \frac{5 \times 6}{4} = 7.5\ \text{kNm}$$

Bending moment at the mid of the span BC

$$= \frac{wl^2}{8} = \frac{2 \times 4^2}{8} = 4\text{kNm}$$

$$a_1x_1 = \left(\frac{1}{2} \times 3 \times 7.5 \times 3 \times \frac{2}{3}\right) + \left(\frac{1}{2} \times 3 \times 7.5\right)\left(3 + \frac{3}{3}\right)$$

$$= 22.5 + 45 = 67.5$$

$$a_2x_2 = \frac{2}{3} \times 4 \times 4 \times 2 = 21.33$$

Now using three moment equation.

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = -\left[\frac{6a_1x_1}{l_1} + \frac{6a_2x_2}{l_2}\right]$$

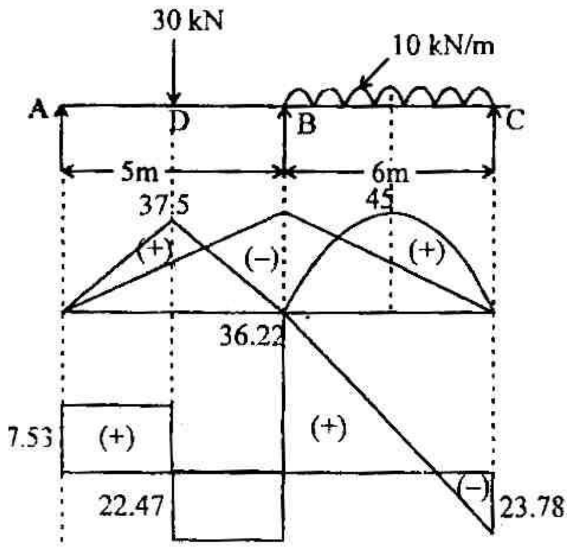
$$\Rightarrow \quad 0 + 2M_B(6 + 4) + 0 = \left[\frac{6 \times 67.5}{6} + \frac{6 \times 21.33}{4}\right]$$

$$\Rightarrow \quad 20M_B = -(67.5 + 31.99) = -99.495$$

$$M_B = -4.97 \text{ Nm}$$

Q.2. A continuous beam ABC is simply supported over support 'A', 'B' and 'C'. the span AB is 5m and BC is 6m. It is subjected to a point load 30 kN at mid span of AB and an UDL 10 kN/m over whole span of BC. Find out the moment at support 'B'. The beam is of uniform cross-section.2013 2(c)

Ans:



Let m_A = Fixing moment at A.

M_B = Fixing moment at B.

M_C = Fixing moment at C.

$$m_D = \frac{wl}{4} = \frac{30 \times 5}{4} = 37.5 \text{ KN.m}$$

bending moment at the mid of the span BC

$$= \frac{wl_2^2}{8} = \frac{10 \times 6^2}{8} = \frac{10 \times 36}{8} = \frac{360}{8} = 45.00 \text{ KN.m}$$

We find that

$$a_1x_1 = \left[\left(\frac{1}{2} \times 2.5 \times 37.5 \times \frac{2 \times 2.5}{3}\right)\right] + \left(\frac{1}{2} \times 2.5 \times 37.5\right)\left(2.5 + \frac{2.5}{3}\right)$$

$$= 78.12 + (46.87 \times 3.33)$$

$$= 78.12 + 156.07 = 234.19$$

Now using three moments equation

$$m_A l_1 + 2m_b(l_1 + l_2) + m_c l_2 = -\left(\frac{6a_1x_1}{l_1} + \frac{6a_2x_2}{l_2}\right)$$

$$0 + 2m_b(5+6) + 0 = -\left(\frac{6\times 234.19}{5} + \frac{6\times 540}{6}\right)$$

$$22m_B = -(281.02 + 540) = -821.02$$

$$m_B = \frac{821.02}{22} = 37.31$$

Shear force diagram

Let R_A = Reaction at A

R_B = Reaction at B

R_C = Reaction at C

Taking moments about B.

$$R_B = R_A \times 5 - (30\times 2.5) - 37/31 = R_A \times 5 - 75$$

$$R_A \times 5 = -37.31 + 75$$

$$R_A = -\frac{37.31+75}{5} = \frac{37.69}{5} = 7.53\text{kN}$$

Similarly $R_C \times 6 - (60\times 3) = -37.31$

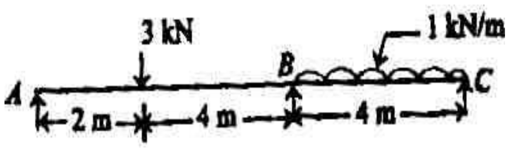
$$R_C \times 6 = -37.31 + (60\times 3)$$

$$R_C = \frac{37.31 + (60\times 3)}{6} = -23.78$$

$$R_B = (30 + 10\times 6) - (7.53 + 23.78) = 90 - 31.31 = 58.69$$

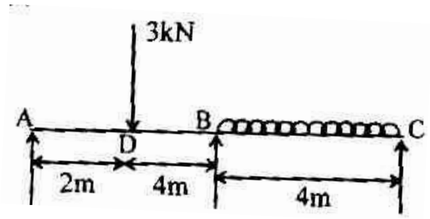
7 MARKS

Q.1. A continuous beam ABC 10m long rests on three support A, B and C at the same level and is loaded as shown in figure.



Determine the moments over the beam and draw the bending moment diagram. Also calculate the reactions at the support and draw shear force diagram using theorem of three moment.

Ans: Applying the theorem of three moments of the span AB and AC



Length of AB = l_1 = 6 m

Length of BC = l_2 = 4m

M_A = Fixing moment of A

M_B = Fixing moment of B

M_C = Fixing moment of C.

Let us consider the beam AB as simply supported so bending moment at D.

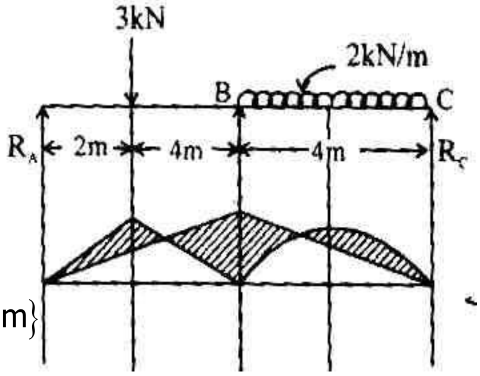
$$M_D = \frac{wab}{l_1} = \frac{3 \times 2 \times 4}{6} = 4 \text{ kN} - \text{m}$$

Bending moment at the mid of the span BC.

$$\frac{Wl_2^2}{8} = \frac{3 \times 4^2}{8} = 6 \text{ kN} - \text{m.}$$

Using three moment equation.

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = \left\{ \frac{6a_1 x_1}{l_1} + \frac{6a_2 x_2}{l_2} \right\}$$



$$a_1 = \frac{1}{2} \times 6 \times 4 = 12\text{m}^2$$

$$x_1 = \frac{6+2}{3} = \frac{8}{3} = 2.67\text{m.}$$

$$x_2 = 2\text{m.}$$

$$\Rightarrow 2M_B(l_1 + l_2) = \left\{ \frac{6 \times 12 \times 2.67}{6} + \frac{6 \times 16 \times 2}{4} \right\}$$

$$= 80.04 \qquad \{\because M_A = M_C = 0 \text{ kN} - \text{m}\}$$

$$\Rightarrow \quad M_B = \frac{-80.04}{2 \times (6+4)} = 4.002 \text{ kN} - \text{m.}$$

Let R_A = Reaction at A

R_B = Reaction at B

R_C = Reaction at C

Taking moment about B.

$M_B = R_A \times 6 - 3 \times 4$

$\Rightarrow$ $4.002 = R_A \times 6 - 12$

$\Rightarrow$ $R_A = \{4.002 + 12\} \times 1/6 = 2.667$ kN

Again $M_B = R_C \times 4 - 4 \times 2$

$\Rightarrow$ $4.002 \times R_C \times 4 - 8$

$\Rightarrow$ $R_C = (4.002 + 8) \times ¼ = 3.0005$ kN.

$\Sigma fy=0$

$\Rightarrow$ $R_A + R_B + R_C = 3 + 4$

$\Rightarrow$ $2.667 + R_B + 3.0005 = 7$

$\Rightarrow$ $R_B = 7 - (2.667 + 3.0005)$

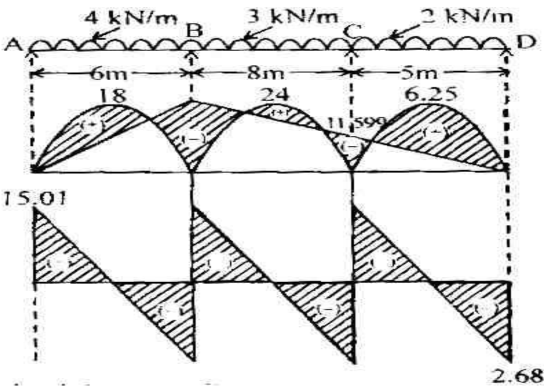
$R_B = 1.3325$ kN.

Q.2. A continuous beam ABCD is simply supported over three spans, such that AB = 6m. BC = 8 m and CD = 5m. It carries UDL of 4 kN/m in span AB, 3 kN/m in span BC and 2 kN/m in span CD. Find the support moments B and C and draw the SF and BM diagrams. 2014 4(c)

Ans:

Applying the theorem of three moments for the spans AB and BC.

$$M_a \times 6 + 2M_B \times (6+8) + M_C \times 8m = \frac{4\times 6^3}{4} + \frac{3\times 8^3}{4}$$



Since A is the simply supported end of the girder.

$M_a = 0$

$28M_B + 8M_C = 216 + 384 = 600$

$14M_B + 4M_C = 300$ ……………………(1)

Consider the span BC and CD.

Applying the theorem of three moments for these spans.

$$M_B \times 8 + 2M_C(8+5) + M_d \times 5 = \frac{3\times 8^3}{4} + \frac{2\times 5^3}{4}$$

Since D is the simply supported end.

$M_d = 0$

$8M_b + 26M_C = 384 + 52.5 = 446.5$

$4M_b + 13M_C = 223.25$ ……………………..(2)

$14M_b + 4M_c = 300$

$4M_b + 13M_c = 223.25$

$10M_b - 9M_c = 76.75$

$10\ M_b - 76.75 + 9M_c$

$$M_B = \frac{76.75 + 9M_C}{10}$$

$$4 \times \frac{76.75 + 9M_C}{10} + 13M_C = 223.25$$

$4M_b + 150\ .79 = 223.25$

$4M_b = 72.45$

$M_b = 18.11$ kN –m

Maximum free bending moment for span AB.

$$= \frac{4 \times 6^2}{8} = 18\text{kN} - \text{m}$$

Maximum free bending moment for span CD

$$= \frac{2 \times 5^2}{8} = 6.25\text{kN} - \text{m}$$

$$\text{B.M at} B = V_a \times 6 - \frac{4 \times 6^2}{2} = 18.11$$

$$V_a = \frac{18.11 + 72}{6}$$

$$V_a = 15.01\text{kN}$$

B.M. at c = $15.01 \times 14 + V_b \times 8 - 4 \times 6 \times 11 - 3 \times 8 \times 4 = 11.599$

$V_b = 11.599 - 210.14 + 264 + 96$

$= 11.599 - 210.14 + 360$

$V_b = 161.459$ kN

$$\text{B.M. at c} = V_d \times 5 - \frac{2 \times 5^2}{2} = -11.599$$

$$V_d \times 5 = -11.599 + 25$$

$V_d = 2.68$kN

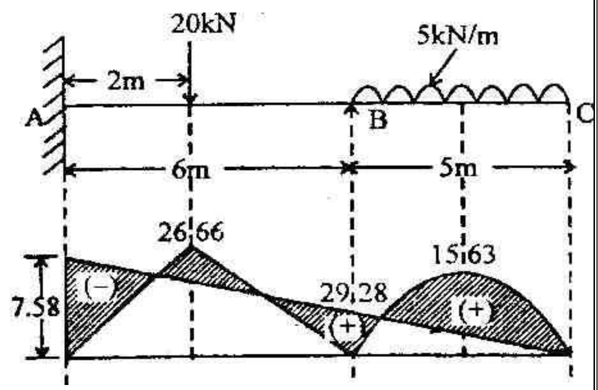
V_c = total load – $(V_a + V_b + V_d)$

$= (4 \times 6 + 3 \times 8 + 2 \times 5) - (15.01+161.459+2.68)$

$= \quad (24 + 24 + 10) - 179.149 = 58 - 179.149 = -121.149$

Q.3. A continuous beam ABC with fixed end at 'A' simply supported over support 'B' and 'C'. the span AB = 6m and BC = 5m. Span AB is subjected to a point load 20 kN at 2m from support 'A' and UDL 5 kn/m over whole span BC. Find out the reactions and moment at supports. Draw the shear force and bending moment diagram for the same. 2016 5(c) 2013 (s)

Ans:



Support moments at A, B and C.

Let m_A = Support moment at A

M_B = Support moment at B

M_C = Support moment at C

Bending moment under the 20 KN load at AB

$$= \frac{wab}{l_1} = \frac{20 \times 2 \times 4}{6} = 26.66 \text{ KN.m}$$

Consider the beam BC as a simply supported beam. Therefore bending moment at the mid of span BC.

$$= \frac{wl_2^2}{8} = \frac{5 \times 5}{8} = 15.63 \text{ KN.m}$$

Geometry of the above bending moment diagram. We find that for the span OA and AB.

$$a_0 x_0 = 0$$

$$a_1 x_1 = \left[\left(\frac{1}{2} \times 26.66 \times 4 \times \frac{2 \times 4}{3}\right) + \left(\frac{1}{2} \times 26.66 \times 2\right)\left(4 + \frac{2}{3}\right)\right]$$

= [142.19+(26.66) (4.67)

=(142.19 + 124.50)=266.69

Similarly for the spans AB and BC.

$$a_1 x_1 = \left[\left(\frac{1}{2} \times 26.66 \times 4 \times \frac{2 \times 4}{3}\right) + \left(\frac{1}{2} \times 26.66 \times 2\right)\left(4 + \frac{2}{3}\right)\right] = 266.69$$

$$a_2 x_2 = \frac{2}{3} \times 15.63 \times 5 \times 2.5 = 130.25$$

$$12m_A + 6m_B = -266.69$$

$$6(2m_A + m_B) = -266.69$$

$$2m_A + m_B = \frac{-266.69}{6} = -44.45..........(i)$$

Now using three moments equation for the span AB and BC.

$$m_A l_1 + 2m_B(l_1 + l_2) + m_C l_2 = -\left[\frac{6a_1x_1}{l_1} + \frac{6a_2x_2}{l_2}\right]$$

$$= m_A \times 6 + 2m_B(6+5) + 0 = -\left[\frac{6\times 266.69}{6} + \frac{6\times 130.25}{5}\right]$$

$6m_A + 22m_B = -[266.69+156.30]$

$6m_A + 22m_B = 422.99$ ……………….. (ii)

Solving equation (i) and (ii)

Equation (i) × 3 = $6m_A + 3m_B = 133.35$

Equation (ii) × 1 = $6m_A + 22m_B = -422.99$

$$\frac{(-)\ (-)\ (-)\quad (+)}{-19m_B = 556.34}$$

$$M_B = \frac{556.34}{19} = -29.28$$

Put the value of m_B in equation (i)

$2\ m_A + (-29.28) = -44.45$

$2\ m_A - 29.28 = -44.453$

$2\ m_A = -44.45 + 29.28$

$= -15.17$ $\quad m_A = \frac{-15.17}{2} = -7.58$

∴ $m_A = -7.58$ Kn-m

$m_B = -29.28$ KN-m

$m_C = 0$

Shear force diagram

Let R_A = Reaction at A

R_B = Reaction at B

R_C = Reaction at C

Taking moment about B

$-29.28 = R_C \times 5 - (25 \times 2.5) - 29.28 = 5R_C - 62.5$

$5\ R_C = -29.28 + 62.5 = 33.22$ $\quad R_C = \frac{33.22}{5} = 6.65$ KN

Now taking moment about A.

$-7.58 = R_B \times 6 - (25 \times 8.5) - (20 \times 2) + 6.65 \times 11$

$-7.58 = 6R_B - 212.5 - 40 + 73.15$

$6R_B = -7.58 + 212.5 + 40 - 73.15 = -80.73 + 252.50 = 171.77$

$$R_B = \frac{171.77}{6} = 28.62KN$$
$$R_A = 20 \times 2 - 28.62$$
$$= 40 - 28.62 = 11.38 \text{ KN}$$

CHAPTER:5

Q.1. Define carry over factor , Stiffness factor 2013 1(j) 2015 4(a)

Ans: Carry over factor: The ratio of moment produced at a joint to the moment applied at the other joint, without displacing , it is called Carry over Factor.

Stiffness Factor: Moment required to rotate end by unit angle, when rotation is permitted at that end, is called stiffness factor.

Q.2. state the Stiffness Factor for a beam fixed at one end & freely supported at the other. 2014 4(a)

Ans: The stiffness factor at fixed end,

K1=4EI/L

The Stiffness Factor at J/S end

K2=3EI/L

Q.3. Find the other State the stiffness factor for a beam fixed at our end & freely supported at. 2017 4(a)

Ans : $K = \frac{4EI}{l}$

(one end fixed s Free support at the other)

Q.4. State the stiffness factor for a beam fixed at one end & freely supported at the other 2017 4 (a)

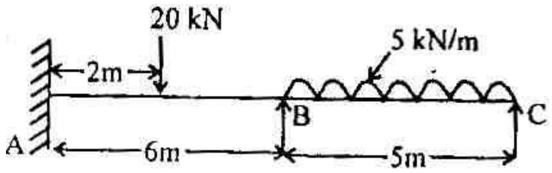
Ans:- $K = \frac{4EI}{L}$ $(One\ end\ Eixed\ \&\ Freely\ supported\ at\ the\ other)$

7 MARKS

Q.1. A continuous beam ABC with fixed end at 'A' S/S over support 'B' and 'C'. the span AB = 6m & BC = 5m, Span AB is subjected to a point load 20 kN at 2m from support 'A' & UDL 5 kN/m over whole Span BC. Find outthe reactions moment of supports by suing moment distribution method.

2016 5(c) 2014 (s)

Ans:



Let us assume the continuous beam ABC to be split up into filxed beams AB, BC.

In span AB, fixing moment at A

$$= \frac{-wab^2}{l^2} = \frac{-20 \times 2 \times (4)^2}{(6)^2} = -17.77 kNm$$

Fixing moment at B,

$$= \frac{-wab^2}{l^2} = \frac{-20 \times 2 \times 4^2}{6} = 8.88 \text{ kNm}$$

In span BC, fixing moment at B

$$= \frac{wl^2}{12} = \frac{-5 \times 5^2}{12} = -10.41 kNm$$

Fixing moment of c $= \frac{+wl^2}{12} 10.41 kNm$

Now let us find out he distribution factor at B. From the geometry of the figure we find that the stiffness factor for BA.

$$k_{BA} = \frac{4EI}{l} = \frac{4E \times I}{6} = \frac{2}{3}EI$$

$$k_{BC} = \frac{4EI}{l} = \frac{4E \times I}{5} = \frac{4}{5}EI$$

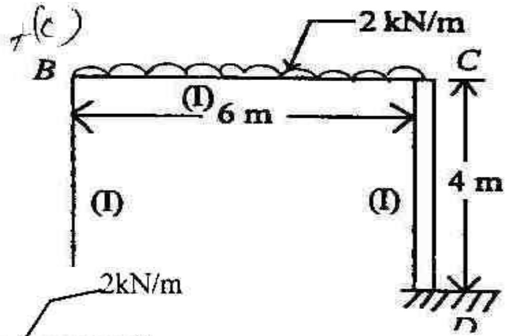
Distribution factor for BA and BC

$$= \frac{\frac{2}{3}EI}{\frac{2}{3}EI + \frac{4}{5}EI} \text{ and } \frac{\frac{4}{5}EI}{\frac{2}{3}EI + \frac{4}{5}EI} = \frac{5EI}{11} \text{ and } \frac{6EI}{11}$$

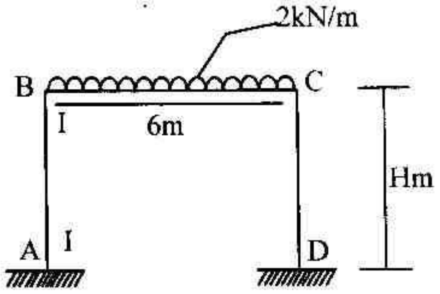
	5/11	6/11		
–17.77	8.88	–10.41	10.41	Fixed end moment
			–10.41	Release c.
		-5.41		Carry over
-17.77	8.88	-15.62	0	Initial moments
	3.06	3.67		Distribute
1.53				
0	0	0	0	Carryover istribute
–16.24	11.94	–11.95	0	Final moment

8 MARKS

Q.1. Analyse the portal frame shown in figure using moment distribution method and draw the bending moment diagram 2015,7(c)



Ans:



Fixed End Moment

$M_{FAB} = M_{FBA} = M_{CD} = M_{FDC} = 0$

$M_{FBC} = \frac{W\ell^2}{12} = \frac{-2 \times 6 \times 6}{12} = -6$ kNm

$M_{FCB} = 6$ kNm.

Distribution Factor :

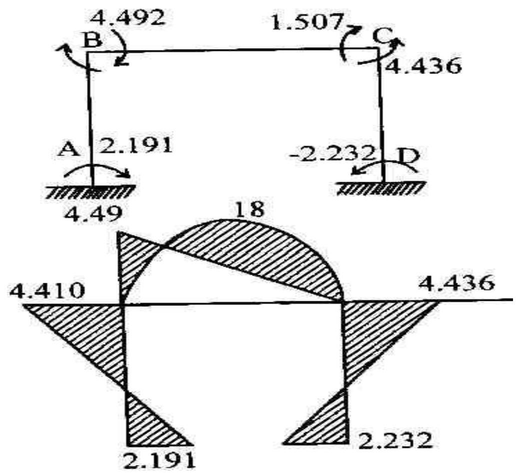
Joints	Member	K	ΣK	Factor
B	BA	4EI/4=EI	SEI/3	$\frac{EI}{SIE/3} = \frac{3}{5} = 0.6$
	BC	$\frac{4EI}{6} = \frac{2}{3}EI$		$\frac{2/3EI}{5/3EI} = \frac{2}{3} \times \frac{3}{5} = 0.4$
C	CB	$\frac{4EI}{6} = \frac{2}{3}EI$		$\frac{2/3EI}{5/3EI} = \frac{2}{5} = 0.4$
	CD	$\frac{4EI}{4} = EI$	SEI/3	EI/5/3 = 3/5 = 0.6

A	BA	BC	CB	CD	D
	0.6	0.4	0.4	0.6	
		– 6	6		
	3.6	2.4	-2.4	- 3.6	
-1.8		– 1.2	1.2		-1.8
	0.72	0.48	-0.48	- 0.72	
0.36		– 0.24	0.24		-0.36
	0.062	0.096	-0.096	-0.144	
0.031		– 0.048	0.048		-0.072
	0.028	0.0192	-0.0192	-0.028	

Free moment diagram of BC parabola with maximum ordinate

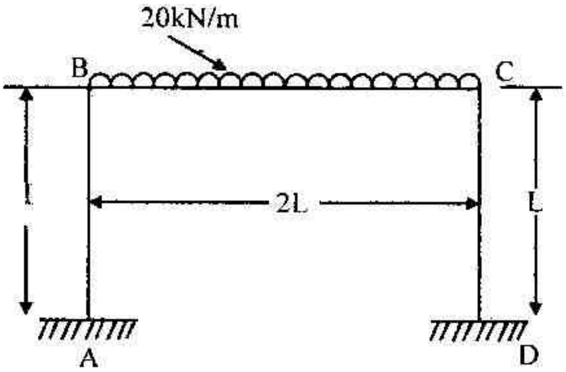
$$\Rightarrow \frac{2 \times 61}{8} = 18\text{kNm}$$

Draw in compression side of member. Difference of these two diagram is BM diagram. Positive and negative are marked upon whether cousing tension/compression on doted side.



Q.2. A rectangular portal frame of uniform flexural rigidity EI, carries a UDL of 20 kN/m as shown in fig. Draw the bending moment diagram and sketch the deflected curve. If L = 4 m and $EI_{AB} = EI_{BC} = EI_{CD}$.

2014(s)



Ans: Given data

Length of AB =4m

Length of Bc = 2 L = 2 × 4 = 8 m^2

Length of CD = 4m

Load on BC (A) = 20 kN/m

$EI_{AB} = EI_{BC} = EI_{CD}$

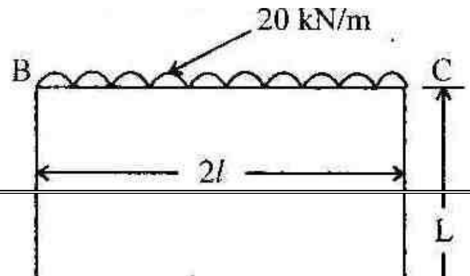
Support Reactions:

Let M_{AB} = Moment at A_a

M_{BA} = Moment at B in span BA.

M_{BC} = Moment at B in span BC.

M_{CB} = Moment at C in span CB.



M_{CB} = Moment at C in span CD.

M_{DC} = Moment at D.

(i) Fixed end moments: Let us assume the frame to be made up of fixed beams AB, BC and CD from the geometry of the figure, we find that fixed end moment at A.

$$M'_{AB} = M'_{BA} = 0$$

$$M'_{BC} = -\frac{wL^2}{12} = -\frac{20\times 8^2}{12} = -106.67kN-m$$

$$M'_{CB} = +\frac{wL^2}{12} = +\frac{20\times 8^2}{12} = +106.67kN-m$$

$$M'_{CB} = M'_{DC} = 0$$

(ii) Slope Deflection Equation : Since frame is fixed at A and D. therefore the slopes i_A and i_D will be equal to zero.

Moment at 'A' in the span AB,

$$M_{AB} = \frac{2EI_{AB}}{\ell}(2i_A + i_B) + M'_{AB}$$

$$= \frac{2EI}{4}(0 + i_B) + 0$$

$$\frac{EI\times i_B}{2}$$

$$M_{BA} = \frac{2EI_{BA}}{\ell}(2i_B + i_A) + M'_{BA}$$

$$= \frac{2EI}{4}(2i_B + i_A) + M'_{BA}$$

$$= \frac{2EI}{4}(2i_B + 0) + 0$$

$$EI\times i_B$$

Moment at 'B' in span BC,

$$M_{BC} = \frac{2E\times I_{BC}}{\ell}(2i_B + i_C) + M'_{BC}$$

$$= \frac{2I\times I}{63}(2i_B + i_C) - 106.67$$

$$= \frac{EI(2i_B + i_C)}{3} - 106.67$$

$$M_{CB} = \frac{2E\times I_{CB}}{\ell}(2i_C + i_B) + M'_{CB}$$

$$= \frac{2EI}{63}(2i_C + i_B) + 106.67$$

Moment at 'C' in span CD.

$$M_{CD} = \frac{2E \times I_{CD}}{\ell}(2i_C + i_D) + M'_{CD}$$
$$= \frac{2EI}{4}(2i_C + 0) + 0$$
$$= EI \times i_C$$
$$M_{DC} = \frac{2EI_{DC}}{\ell}(2i_D + i_C) + M'_{DC}$$
$$= \frac{2EI}{4}(0 + i_C) + 0$$
$$= \frac{EI \times i_C}{2}$$

EXAMPLE 19.10. *A horizontal steel girder having uniform cross-section is 14 m long and is simply supported at its ends. It carries two concentrated loads as shown in Fig. 19.10.*

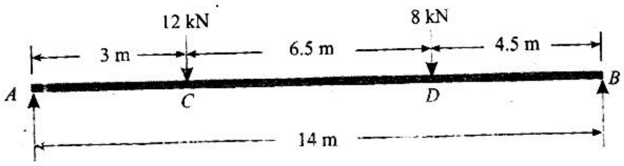


Fig. 19.10

Calculate the deflections of the beam under the loads C and D. Take E = 200 GPa and I = 160×10^6 mm^4.

SOLUTION. Given: Span (l) = 14 m = 14×10^3 mm ; Load at C (W_1) = 12 kN = 12×10^3 N ; Load at D (W_2) = 8 kN = 8×10^3 N ; Modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2 and moment of inertia (I) = 160×10^6 mm^4.

Taking moments about A and equating the same,

$$R_B \times 14 = (12 \times 3) + (8 \times 9.5) = 112$$

$$\therefore \quad R_B = \frac{112}{14} = 8 \text{ kN} = 8 \times 10^3 \text{ N}$$

and $$R_A = (12 + 8) - 8 = 12 \text{ kN} = 12 \times 10^3 \text{ N}$$

∴ Now taking A as the origin and using Macaulay's method, the bending moment at any section X at a distance x from A,

$$EI \frac{d^2y}{dx^2} = -(12 \times 10^3)\, x + \Big|\ (12 \times 10^3) \times [x - (3 \times 10^3)] + (8 \times 10^3) \times [x - (9.5 \times 10^3)]$$

Integrating the above equation,

$$EI \frac{dy}{dx} = -(12 \times 10^3) \frac{x^2}{2} + C_1 \Big|\ + (12 \times 10^3) \times \frac{[x - (3 \times 10^3)]^2}{2} + (8 \times 10^3) \times \frac{[x - (9.5 \times 10^3)]^2}{2}$$

$$= -(6 \times 10^3)\, x^2 + C_1 \Big|\ + (6 \times 10^3) \times [x - (3 \times 10^3)]^2 + (4 \times 10^3) \times [x - (9.5 \times 10^3)]$$

Integrating the above equation once again,

$$EI \cdot y = -(6 \times 10^3) \times \frac{x^3}{3} + C_1 x + C_2 + \Big|\ (6 \times 10^3) \times \frac{[x - (3 \times 10^3)]^3}{3} + (4 \times 10^3) \times \frac{[x - (9.5 \times 10^3)]^3}{3}$$

$$= (2 \times 10^3)\, x^3 + C_1 x + C^2 \Big|\ + (2 \times 10^3)\, [x - (3 \times 10^3)]^3 + \frac{4 \times 10^3}{3} \times (x - (9.5 \times 10^3)]^3 \quad \ldots$$

We know that when $x = 0$, then $y = 0$. Therefore $C_2 = 0$. And when $x = (14 \times 10^3)$ mm, then y

Therefore

$$0 = -(2 \times 10^3) \times (14 \times 10^3)^3 + C_1 \times (14 \times 10^3) + (2 \times 10^3) \times [(14 \times 10^3) - (3 \times 10^3)]^3 + \frac{4 \times 10^3}{3} \times [(14 \times 10^3)\ \ (9.5 \times 10^3)]^3$$

$$= -(5488 \times 10^{12}) + (14 \times 10^3)\, C_1 + (2662 \times 10^{12}) + 121.5 \times 10$$

$$= -(2704.5 \times 10^{12}) + (14 \times 10^3)\, C_1$$

$$\therefore \quad C_1 = \frac{2704.5 \times 10^{12}}{14 \times 10^3} = 193.2 \times 10^9$$

Substituting the value of C_1 equal to 193.2×10^9 and $C_2 = 0$ in equation (*ii*),

$$EIy = -2 \times 10^3 x^3 + 193.2 \times 10^9 x \;\vdots\; + 2 \times 10^3 [x - (3 \times 10^3)]3$$

$$\vdots + \frac{4 \times 10^3}{3} \times [x - (9.5 \times 10^3)]^3 \quad \ldots(iv)$$

Now for deflection under the 12 kN load, substituting $x = 3$ m (or 3×10^3 mm) in equation (*iii*) up to the first dotted line only,

$$EIy_C = -2 \times 10^3 \times (3 \times 10^3)^3 + 193.2 \times 10^9 \times (3 \times 10^3)$$

$$= -(54 \times 10^{12}) + (579.6 \times 10^{12}) = 525.6 \times 10^{12}$$

$$\therefore \quad y_C = \frac{525.6 \times 10^{12}}{EI} = \frac{525.6 \times 10^{12}}{(200 \times 10^3) \times (160 \times 10^6)} = \mathbf{16.4\ mm} \quad \textbf{Ans}$$

Substituting the value of C_1 again in equation (*iv*) and $C_2 = 0$,

$$EI \cdot y = -\frac{Wbx^3}{6l} + \frac{Wbx}{6l}(l^2 - b^2) \;\vdots\; + \frac{W(x-a)^3}{6}$$

This is the required equation for deflection at any point. For deflection in *AC*, consider the equation up to the dotted line only,

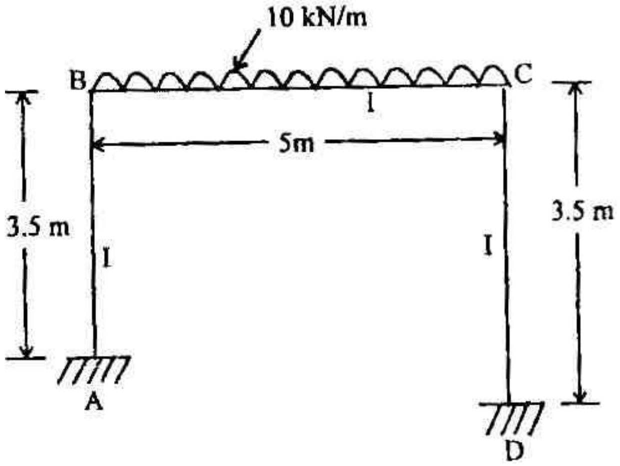
$$EI \cdot y = -\frac{Wbx^3}{6l} + \frac{Wbx}{6l}(l^2 - b^2) = \frac{Wbx}{6l}(l^2 - b^2 - x^2)$$

$$\therefore \quad y = \frac{Wbx}{6EIl}(l^2 - b^2 - x^2) \quad \ldots \text{(As before)}$$

Ch-5 Ans

Q.3. Analyze a symmetrical rectangular portal frame using deflection method of horizontal span 5m subjected to an UDL 10 kN/m over whole span and height of 3.5 m. 2013, (7)

Ans:



The ends A and D of the frame ABCD are fixed and therefore

$\theta_A = \theta_D = 0$

As the portal frame is symmetrical and loaded symmetrically rotation $\theta_3 = \theta_c$ and there will be no way i.e. $\delta = 0$.

The fixed moments are :

$$M_{BC} = \frac{-10 \times 5 \times 5}{12} = -20.83 kNm$$

$$M_{CB} = +20.83 kNm$$

The slope deflection equations in terms of unknown are :

$$M_{AB} = \frac{2EI}{3.5}(\theta + \theta_B - 0) = \frac{2EI\theta_B}{3.5}$$

$$M_{BA} = \frac{2EI}{3.5}(\theta + 2\theta_B) = \frac{4EI\theta_B}{3.5}$$

$$M_{BC} = \frac{2EI}{5}(2\theta_3 + \theta_B) - 20.83$$

$$= \frac{-2}{5}EI\theta_B + 20.83$$

For equilibrium the sum of the moments at joint B is zero.

$$\therefore M_{AB} + M_{BC} = 0$$

$$\Rightarrow \frac{2EI\theta_B}{3.5} + \frac{2}{5}EI\theta_B - 20.83 = 0$$

$$\Rightarrow 2EI\theta_B\left(\frac{1}{3.5} + \frac{1}{5}\right) - 20.83 = 0$$

$$\Rightarrow 2EI\theta_B \times 0.48 = 20.83$$

$$\theta_B = \frac{21.69}{EI}$$

$$\therefore M_{AB} = \frac{2EI\theta_B}{3.5} = \frac{2EI}{3.5} \times \frac{21.69}{EI} = 12.39kNm$$

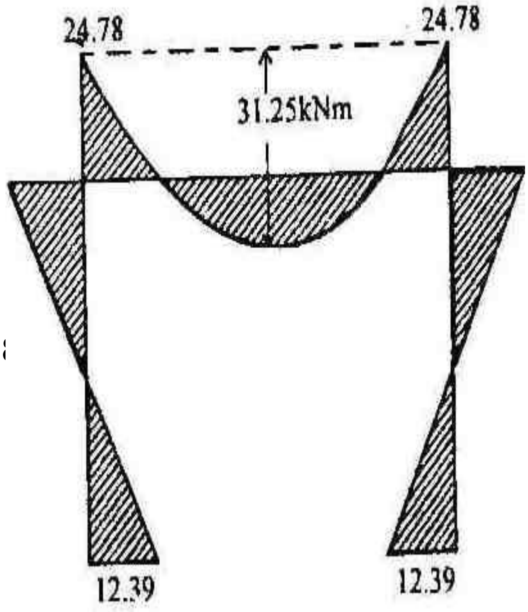
$$\therefore M_{BA} = \frac{4EI\theta_B}{3.5} = \frac{2EI}{3.5} \times \frac{21.69}{EI} = 24.78kNm$$

$$\therefore M_{BC} = \frac{2}{5}EI\theta_B - 20.83$$

$$= \left(\frac{2}{5} \times EI \times \frac{21.69}{EI}\right) - 20.83 = -12.15kNm$$

$$M_{CB} = \frac{-2}{5}EI\theta_B + 20.83 = \left(\frac{-2}{5} \times EI \times \frac{21.69}{EI}\right) + 20.8$$

$$= 12.15kNm$$



iii) Equilibrium equations: Since the joints B and C are in equilibrium, so first equation $M_{BA} + M_{BC}$ equal to zero.

$(EI \times i_B) + [EI (2 i_B + i_C) - 106.67] = 0$

$(EI \times i_B) + (2EI \times i_B) + (EI \times i_C) - 106.67 = 0$

$3 (EI \times i_B) + (EI \times i_C) = 106.67.$

Now equation $M_{CB} + M_{CD} = 0$

$[EI(2i_C + i_B) + 106.67] + (EI \times i_C) = 0$

$2(EI \times i_C) + (EI \times i_B) + 106.67 + (EI \times i_C) = 0$

$3(EI \times i_C) + (EI \times i_B) + 106.67 = 0$

By symmetry, $i_B = i_C$, substituting these values, we get $2EI \times i_B = 106.67$

$$EI \times i_B = \frac{106.67}{2} = 53.335$$

$$2EI \times i_C = -106.67$$

$$EI \times i_C = -\frac{106.67}{2} = -53.335$$

Final moments :

$$M_{AB} = \frac{EI \times i_B}{2}$$

$$= \frac{53.335}{2} = 26.667 \text{ kN} - \text{m.}$$

$$M_{BA} = EI \times i_B = 53.335 \text{ kN} - \text{m.}$$

$$M_{BC} = \frac{EI(i_B + i_C)}{3} = 106.67$$

$$= \frac{(2EIi_B) + (EIi_C)}{3} - 106.67$$

$$= \frac{106.67 + (-53.335)}{3} - 106.67$$

$$= -88.89 \text{ kN} - \text{m.}$$

$$M_{CB} = \frac{EI(2i_C + i_B)}{2} + 106.67$$

$$= \frac{2EIi_C + EIi_B}{3} + 106.67$$

$$= \frac{106.67 + 53.335}{2} + 106.67$$

$$= 80.00 \text{ kN} - \text{m.}$$

$$M_{CD} = EI \times i_C$$

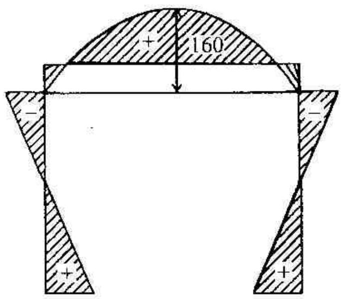
$$= -53.335 \text{ kN} - \text{m.}$$

$$M_{DC} = \frac{EI \times i_C}{2}$$

$$\frac{-53.335}{2} = 26.667 \text{ kN} - \text{m.}$$

The bending moment at the mid of the span BC, buy considering it as a simply supported beam.

$$= \frac{w\ell^2}{8} = \frac{20 \times 8^2}{8} = 160 \text{ kN} - \text{m.}$$



CHAPTER:6

2 MARKS

Q.1. In case of a column whose both ends are hinged , what will be its equivalent length. 2015, 4(a)

Ans: In Case of a column whose both ends are hinged. Then its equivalent length will be same as its actual length le = l

5 MARKS

Q.1. A steel rod 5m long and 40 mm dia. is used as a column with one end fixed & other free. Determine the cripping load by Euler's formula. Take l as 200 GPa. 2016, 7(b)

Ans: Given data l = 5 m = 5 × 10^3 mm

D = 40 mm

L = 200 GPa = 200 × 10^3 N/mm^2

One end fixed other free le = 2l

Bu Euler's formula

$$P_{Eulers} = \frac{\pi^2 EI}{le^2}$$

$$I = \frac{\pi d^4}{64} = \frac{\pi \times (40)^4}{64} = 125663.7 mm^4$$

$$P_{Eulers} = \frac{\pi^2 \times 200 \times 10^3 \times 125663.7}{(2 \times 5 \times 10^3)^2}$$

$$= 24805 N$$

$$= 24.85 N$$

Q.2. State different end conditions of column and write down the relation between equivalent length and actual length in each case. 2015, 6(b)

Ans: In actual practice there are a number of end conditions, for columns. But we shall study the Euler's column theory on the following four types of end conditions, which are important from the subject point of view.

Types of End Conditions	Relation between equivalent Length (L_e) & actual length (ℓ)
1. Both ends hinged	$L_e = \ell$
2. One end fixed and the other free	$L_e = 2\ell$
3. Both ends fixed	$L_e = \ell/2$
4. One end fixed and the other hinged	$L_e = \ell/\sqrt{2}$

Q3.- State different end conditions of calumn and write down the relation between equivalent length , actual length in each case. 20176(b)

Ans

End conditions

1) Both ends hinged
2) One end fixed and the other free
3) Both ends fixed
4) One end fixed and the other hinged

Relation between equivalent length (Le) and actual length (L)

Le=l

Le=2l

Le=$\frac{l}{2}$

L2=$\frac{l}{\sqrt{2}}$

CHAPTER:7

2 MARKS

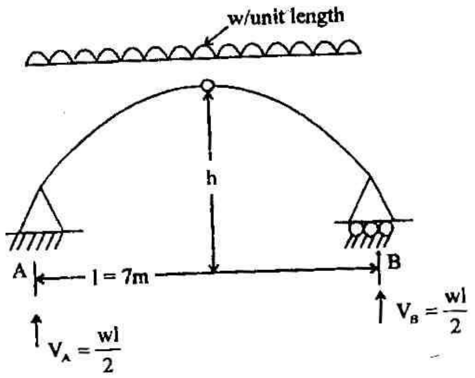
Q.1. What do you mean by three hinged arch ? 2013, 1(d), 2016, 6(a)

Ans: Vector diagram L Diagram showing the magnitude of forces along with direction is called vector dig. Polar diagram: Diagram showing magnitude of forces is called polar diagram.

5 MARKS

Q.1. A three hinged symmetrical arch of span 7m is subjected on UDL of w over whole span. Draw the S.F. & B.M.D. for the arch 2013, 2(f)

Ans:



Let the rise of the arch = hm , Due to symmetry

$V_A = V_B = ½ \times$ total load.

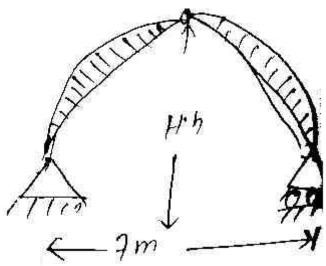
$$= \frac{1}{2} \times w.L = \frac{w \times 7}{2} = 3.5w$$

Taking moment about c, we get

$$O = V_A.\frac{1}{2} - Hh - w\frac{L}{2}.\frac{L}{4}$$

$$= \frac{WL}{2} \times \frac{L}{2} - Hh - \frac{wL^2}{8}$$

$$\therefore H = \frac{wL^2}{8} = \frac{49w}{8}$$



At any section distance x from A

$$M = V_A x - Hy - \frac{wL^2}{2}$$

But in parabolic arxh $y = \frac{4hx(L-x)}{L^2}$

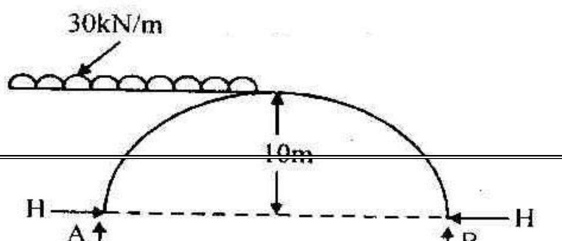
$$\therefore M = \frac{wL}{2}x - \frac{wL^2}{8}.\frac{4hx(L-x)}{L^2} - \frac{wL^2}{2}$$

$$= \frac{wLx}{2} - \frac{w}{2}x(L-x) - \frac{wL^2}{2} = 0$$

CHAPTER-7

5 MARKS

Q.1. A three-hinged parabolic arch of span 40m, and rise 10 m is carrying a UDL as shown in the fig. Find the horizontal thrust at the springing. 2014,5(b)



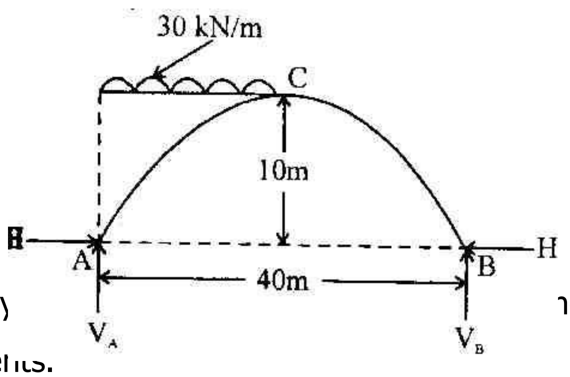
Ans: Given data,

Span(l) = 40m and central rise (y_C) = 10m.

H = Horizontal thrust at the springing.

V_A = Vertical reaction at A.

V_B = Vertical reaction at B.



Vertical reaction V_B at B can be calculated by ... ng anti-clockwise moments with clockwise moments.

$$V_B \times 40 = (30 \times 20) \times 10 = 6000$$

$$V_B = \frac{6000}{40} = 150\text{kN}.$$

The beam moment at C due to external loading

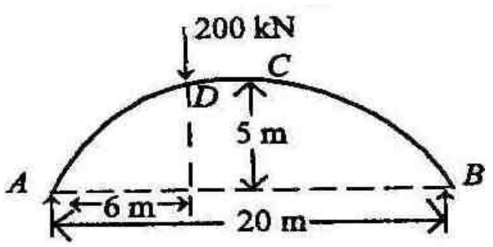
$\mu_C = V_B \times 20 = 150 \times 20 = 3000$ kN-m.

Horizontal thrust,

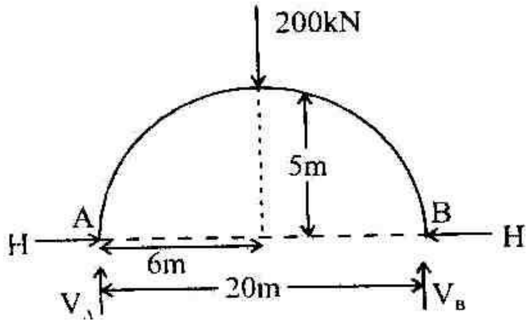
$$H = \frac{\mu_c}{y_c} = \frac{3000}{10} = 300\text{kN}$$

Q.2.

Find out the reaction at A and B and draw the bending moment diagram for the parabolic arch as shown in figure. 2015, 6(c)



Ans:



$\sum M_B = 0$

$\Rightarrow V_A \times 20 - 200(20-6) = 0$

$\Rightarrow V_A \times 20 - 2800 = 0$

$\Rightarrow V_A = \frac{2800}{20} = 140kN$

$\Rightarrow V_A + V_B = 200kN$

$\Rightarrow V_B = 200 - 140 = 60kN$

Bending moment diagram is a triangle with maximum ordinate at load point.

$$\Rightarrow \frac{200 \times 6(20-6)}{20} = \frac{200 \times 6 \times 14}{20} = 84kNm$$

At mid-span the net bending moment is zero. Ordinate of the beam moment diagram is zero. Ordinate of bending moment diagram at mid span is :

$\frac{84 \times 10}{14} = 60kN$

$M_c = 0(\text{In Arch}), Hh = 60kN$

$H = \frac{60}{h} = \frac{60}{5} = 12kN.$

Parabola drawn with central ordinate equal to 60 kNm.

$$Y_{BM} = Y_{\text{Bending Moment}} = Hy = H \times \frac{4hx(L-x)}{\ell^2}$$